The Persistence of False Paradigms in Low-Power Sciences

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ABSTRACT: We develop a model describing how false paradigms may persist, hindering scientific progress. The model features two paradigms, one describing reality better than the other. Tenured scientists display homophily: they favor tenure candidates who adhere to their paradigm. As in statistics, power is the probability (absent any bias) of denying tenure to scientists adhering to the false paradigm. The model shows that because of homophily, when power is low, the false paradigm may prevail. Then only an increase in power can ignite convergence to the true paradigm. Historical case studies suggest that low power comes either from lack of empirical evidence, or from reluctance to base tenure decisions on available evidence.

SIGNIFICANCE STATEMENT: It is believed that a lack of experimental evidence (typical in social science) slows but does not prevent the adoption of true theories. We evaluate this belief using a model of scientific research and promotion in which tenured scientists are slightly biased toward tenure candidates with similar beliefs. We find that when a science lacks evidence to discriminate between theories, or when tenure decisions do not rely on available evidence, true theories may not be adopted. The nonadoption of heliocentric theory in the 16th century, the persistence of bloodletting in the 19th century, the nonadoption of underconsumption theory in the early 20th century, and the persistence of radical mastectomy in the 20th century illustrate such risk.

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Introduction

Friedman (1953) argued that while the social sciences lack the experimental evidence available in the physical sciences, this characteristic only slows—but does not prevent—false theories from being "weeded out." This paper develops a model of science and revisits this argument. We show that when scientists are unbiased, all sciences indeed always converge to the truth, even if they lack empirical evidence to test theories. But there is also homophily; and with homophily in the tenure process, sciences lacking empirical evidence may never converge to the truth. Thus, a lack of experimental evidence may do more than just slow down scientific progress: it may allow worse theories to prevail over better ones.

In modeling science, we follow Kuhn (1996). We introduce two paradigms, one giving a better description of the world than the other. Scientists adhere to one or the other paradigm. Scientific inquiry occurs in an environment fashioned after the tenure system in academia: advisees trained by tenured scientists hope to become tenured scientists themselves. The strength of a paradigm is measured by the fraction of tenured scientists adhering to it; a paradigm is weeded out as more and more tenured scientists adhere to the other paradigm.

The power and significance of a science are defined using concepts from statistics. Power is the probability of rejecting a worse-paradigm tenure candidate—just as the power of a statistical test is the probability of rejecting a false null hypothesis. Similarly, significance is the probability of rejecting a better-paradigm tenure candidate. How well a science distinguishes between true and false paradigms is measured by the difference between power and significance. Low power is problematic because it reduces this difference, such that false-paradigm scientists are barely more likely to be denied tenure than true-paradigm scientists.

Our central assumption is that scientists have homophilous bias: in the decision to grant tenure, they favor scientists adhering to their paradigm and discriminate against those adhering to the other paradigm. Homophilous bias has been widely documented (McPherson, Smith-Lovin, and Cook 2001; Dovidio and Gaertner 2010); in fact, even minimal divisions, such as telling groups of boys whether they prefer Klee or Kandinsky, can create strong biases (Tajfel 1970). Homophilous bias has been observed in science: people favor others from the same school of thought at every level of academic evaluation—hiring, conference invitations, peer review, tenure evaluations, and awards of grants and honors (Lamont 2009; Mahoney 1977; Travis and Collins 1991). This bias could also explain publication patterns in scientific fields after the early death of a "star": collaborators of the star publish much less, whereas noncollaborators—many of them new entrants to the field—publish much more, and are highly cited (Azoulay, Fons-Rosen, and Graff Zivin 2015).

Our main result is that a small amount of homophilous bias makes a big difference regarding which paradigm prevails. Without bias, irrespective of power, science always converges to the better paradigm. With homophilous bias, things change: when power is high, science still always converges to the better paradigm; but when power is low, science may converge to the worse paradigm if few scientists initially believe in the better paradigm. Thus, low power does more than just slow down scientific progress: it generates different dynamics, according to which the worse paradigm may prevail. Once a worse paradigm is entrenched, convergence to a better paradigm may only occur if power increases.

That the worse paradigm may prevail at all may be surprising since neither is it more fruitful for research nor do its adherents have greater bias. The worse paradigm may prevail nonetheless because once it has many adherents, a tenure candidate is highly likely to be evaluated by a worse-paradigm scientist. Due to scientists' homophilous bias, this situation advantages worse-paradigm candidates, to the point that they may become more likely to receive tenure than better-paradigm candidates. Then, the number of tenured scientists believing in the worse paradigm grows faster than the number of tenured scientists believing in the better paradigm: science moves away from the truth.

Case studies from the history of astronomy, medicine, and economics illustrate the two patterns in our model: entrenchment of inferior paradigms when power is low; and convergence toward superior paradigms once power increases—what Kuhn calls scientific revolutions. In these case studies, power is determined both by the data and methods available to test theories and by the norms regarding the appropriate criteria to use in tenure decisions.

Our paper contributes to an emerging literature on the theoretical underpinnings of scientific progress. This literature explores how false claims may be accepted as facts if there is a publication bias in favor of positive results (Nissen et al. 2016); how consensus about theories emerge if scientists value conformity (Brock and Durlauf 1999); how new paradigms are created (Bramoulle and Saint-Paul 2010); and how religion influences scientific progress (Benabou, Ticchi, and Vindigni 2015).

The population dynamics obtained in this paper are analogous to those that arose when entomologists at the University of Chicago placed populations of two species of flour beetle into jars in the 1950s and 1960s. The entomologists expected that the species more fit to the environment—the species with the higher carrying capacity—would always dominate. But that is not what happened; instead, one or the other species always vanished (Neyman, Park, and Scott 1956). On close inspection, the reason was determined: beetles eat their own species' eggs, but are yet more prone to eating the eggs of other species (Park et al. 1965). In the flour jars, as in our model, when there is egg-eating bias, two species cannot coexist, and the species doing better in isolation does not necessarily survive in competition.

Model of Science

There are two distinct paradigms: Better and Worse. The Better paradigm gives a more correct description of reality than the Worse, but nobody knows it. Each scientist adheres either to the Better or to the Worse paradigm. At time t, $B(t)$ tenured scientists believe in the Better paradigm, and $W(t)$ believe in the Worse paradigm. The fraction of tenured scientists who believe in the Better paradigm is

$$\sigma(t) = \frac{B(t)}{B(t) + W(t)}. \tag{1}$$

Knowledge is embodied by established scientists, so the strength of a paradigm is measured by the fraction of its adherents among tenured scientists. Since the Better paradigm offers a superior description of the world, knowledge has made progress when $\sigma(t)$ becomes closer to 1.

Scientists' research is based on their respective paradigm: they use their paradigm to explain empirical observations, to guide empirical investigations, and to make theoretical predictions. The quality of a scientist's research is then partially determined by her paradigm. The Better paradigm explains more observations, generates more fruitful empirical investigations, makes more accurate predictions, and can be more easily adjusted to resolve anomalies. Thus, on average, Better scientists produce research of higher quality than Worse scientists. Because no paradigm perfectly describes the real world, however, there is some randomness in research quality. The research process brings additional uncertainty to research quality: empirical observations are difficult to obtain and subject to measurement error; theoretical explanations and predictions are hard to formulate; and scientists vary in skill, effort, and imagination. So research quality is noisy and only partially determined by the underlying paradigm.

The initial numbers of scientists adhering to the Better and Worse paradigms are $B(0)$ and $W(0)$. We do not model early adoption, at which stage tenured scientists defect from the existing paradigm and spontaneously adhere to the newly invented paradigm: we take $B(0)$ and $W(0)$ as given. Our focus is on the competition between Better and Worse paradigms through the tenure system once early adopters start teaching students about their paradigm.

Tenured scientists train advisees at rate $\lambda > 0$. Advisees adhere to the same paradigm as their advisor, and during their entire career, scientists adhere to the same paradigm. Once an advisee is trained, she becomes an untenured scientist and produces research articulated around her paradigm. Then she is brought up for tenure. The evaluator of a tenure candidate is randomly chosen from the population of tenured scientists; thus, the candidate is evaluated by a Better scientist with probability $\sigma(t)$ and by a Worse scientist with probability $1 - \sigma(t)$. If she receives tenure, she continues doing research, advises students, and retires at rate δ. If she does not receive tenure, she quits academia.

Let's first consider the case with no homophilous bias in the grant of tenure. Tenure is entirely determined by the quality of research: all candidates whose research quality is above a certain threshold are granted tenure; all others are denied. As research quality is noisy, not all Better candidates are granted tenure and not all Worse candidates are denied. Nevertheless, Better candidates tend to produce better research than Worse candidates, so they are less likely to be denied tenure. For this reason we assume that α, the probability of denying tenure to a Better candidate, is lower than $1 - \beta$, the probability of denying tenure to a Worse candidate: $\alpha < 1 - \beta$.

Although the tenure test assesses many attributes of the tenure candidate beside the correctness of her paradigm, the evaluation of a tenure case can be interpreted as a statistical test in which (a) the null hypothesis is that the tenure candidate believes in the more correct paradigm; (b) the alternative hypothesis is that the tenure candidate believes in the less correct paradigm; (c) the null hypothesis is rejected when tenure is denied; and (d) the null hypothesis is accepted when tenure is granted. Then α is the probability of rejecting the null even though the null is valid, and $1 - \beta$ is the probability of rejecting the null when indeed the null is invalid. Using the analogy with statistics, we introduce two concepts:

DEFINITION 1: *The significance of a science is the probability α of denying tenure to a Better scientist when there is no homophilous bias. The power of a science is the probability $1 - \beta$ of denying tenure to a Worse scientist when there is no homophilous bias.*

The difference between power and significance measures the gap between the tenure probabilities of Better and Worse scientists when there is no bias; it therefore captures the ability of a scientific field to distinguish between better and worse paradigms. The highest difference is achieved if no Worse scientists receive tenure: $1 - \beta = 1$. The lowest difference occurs if all scientists receive tenure with the same probability: $1 - \beta \to \alpha$. Between these two extremes, a Better scientist is more likely to receive tenure than a Worse scientist, but Worse scientists have some chance of receiving tenure: $\alpha < 1 - \beta < 1$.

Let's now add in the assumption that scientists are biased in favor of those who belong to their paradigm and against those who belong to the alternative paradigm. The agreement or disagreement of belief between the untenured candidate and her tenured evaluator affects tenure decisions: a Better evaluator grants tenure to a Better candidate with higher probability than a Worse evaluator; a Worse evaluator grants tenure to a Worse candidate with higher probability than a Better evaluator. Formally, to the probabilities α and β we add the homophilous bias

$\epsilon \in [0,1]$. With bias, the tenure decisions are as follows. A Better evaluator denies tenure to a Better scientist with reduced probability $(1-\epsilon)\,\alpha$. A Worse evaluator denies tenure to a Better scientist with increased probability $(1+\epsilon)\,\alpha$. A Better evaluator grants tenure to a Worse scientist with reduced probability $(1-\epsilon)\,\beta$. Finally, a Worse evaluator grants tenure to a Worse scientist with increased probability $(1+\epsilon)\,\beta$. (To ensure that all probabilities remain in $[0,1]$, we add the restrictions that $\alpha \leq 1/(1+\epsilon)$ and $\beta \leq 1/(1+\epsilon)$.)

Tenure probabilities now depend not only on α and β but also on ϵ and σ. With evaluators drawn randomly from the population of tenured scientists, a tenure candidate is evaluated by a Better evaluator with probability σ and by a Worse evaluator with probability $1-\sigma$. Hence, a Better candidate is denied tenure with probability

$$\hat{\alpha}(\sigma) = \sigma\,(1-\epsilon)\,\alpha + (1-\sigma)\,(1+\epsilon)\,\alpha = (1+\epsilon)\,\alpha - 2\epsilon\alpha\sigma.$$

Similarly, a Worse candidate is granted tenure with probability

$$\hat{\beta}(\sigma) = \sigma\,(1-\epsilon)\,\beta + (1-\sigma)\,(1+\epsilon)\,\beta = (1+\epsilon)\,\beta - 2\epsilon\beta\sigma.$$

Using again the analogy with statistics, $\hat{\alpha}$ is the probability of type I error (false-positive finding) and $\hat{\beta}$ the probability of type II error (false-negative finding).

We measure the difference between the tenure probabilities of Better and Worse scientists with the Youden (1950) index:

DEFINITION 2: *The Youden index of a science is*

$$J(\sigma) = 1 - \hat{\alpha}(\sigma) - \hat{\beta}(\sigma) = 1 - (1+\epsilon)\,(\alpha+\beta) + 2\epsilon\,(\alpha+\beta)\,\sigma. \tag{2}$$

The Youden index is defined as in statistics: one minus the probability of type I error minus the probability of type II error. The Youden index is linearly increasing in $\sigma \in [0,1]$, from $J(0) = 1-(1+\epsilon)\,(\alpha+\beta)$ to $J(1) = 1-(1-\epsilon)\,(\alpha+\beta)$. It is minimized when $\sigma = 0$ because then all tenured scientists believe in the Worse paradigm, and they are biased against Better tenure candidates and in favor of Worse tenure candidates. It is maximized when $\sigma = 1$ because then all tenured scientists believe in the Better paradigm, and they are biased in favor of Better candidates and against Worse candidates. Thus if $1-\beta \geq \alpha + \epsilon/(1+\epsilon)$, the Youden index is positive for all $\sigma \in (0,1)$. And if $1-\beta < \alpha + \epsilon/(1+\epsilon)$, the Youden index is negative for $\sigma < \sigma^*$, zero at $\sigma = \sigma^*$, and positive for $\sigma > \sigma^*$, where $\sigma^* \in (0,1/2)$ is defined by

$$\sigma^* = \frac{1}{2}\left[1 - \frac{1-(\alpha+\beta)}{\alpha+\beta}\cdot\frac{1}{\epsilon}\right]; \tag{3}$$

σ^* will be the key threshold in the dynamics of $\sigma(t)$.

Overall, the model gives a faithful representation of what Kuhn (1996) calls revolutionary science—in contrast to normal science. Most of the time, scientists engage in normal science: they work within an accepted paradigm, revealed in textbooks and lectures. They use the paradigm to determine important facts; match the paradigm with these facts; and further articulate the paradigm to improve its fit with nature. Our model focuses instead on periods of revolutionary science: when two paradigms compete. Such phases of science arise in response to discovery of anomalies inconsistent with the old paradigm. In these phases, the decision to reject one paradigm is also the decision to accept another.

In our model, scientific knowledge is embodied by established scientists, with the strength of a paradigm indexed by the fraction of scientists adhering to it. This representation accords with Kuhn, who says that a paradigm prevails only after its acceptance by the scientific community. Scientific revolutions are battles of old versus new paradigms for the allegiance of that community.

According to Kuhn, new adherents to a paradigm are not converts from the old one, but are freshly minted scientists. For instance, Kuhn approvingly quotes Planck: "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it" (Kuhn 1996, p. 151). Similarly, in our model, tenured scientists do not switch allegiance, and the ranks of a paradigm grow over time from newly tenured scientists.

Kuhn also emphasizes "resistance" against new paradigms by the adherents of the old paradigm. Such resistance is represented in our model by homophilous bias.

Population Dynamics

We now study the dynamics of the population of scientists to determine under which conditions each paradigm—Better or Worse—eventually prevails.

The population dynamics are the outcome of a horse race regarding which type of scientist is growing at the faster rate. Indeed, differentiating (1) with respect to t, we find that the fraction of Better scientists in the population of tenured scientists evolves according to

$$\dot{\sigma}(t) = \sigma(t)\,[1-\sigma(t)]\left[g^B(t) - g^W(t)\right], \tag{4}$$

where $g^B(t) = \dot{B}(t)/B(t)$ is the growth rate of Better tenured scientists, and $g^W(t) = \dot{W}(t)/W(t)$ is the growth rate of Worse tenured scientists. The growth rate $g^B(t)$ is simply

$$g^B(t) = \lambda\,[1 - \hat{\alpha}(\sigma(t))] - \delta.$$

The first term reflects that Better scientists train advisees at rate λ; and with probability $1-\hat{\alpha}(\sigma(t))$ the advisees are granted tenure. The second term reflects that tenured scientists retire at rate δ. Similarly,

$$g^{W}(t) = \lambda\hat{\beta}(\sigma(t)) - \delta.$$

The first term reflects that Worse scientists train advisees at rate λ; and with probability $\hat{\beta}(\sigma(t))$ such advisees are granted tenure. The second term reflects retirement. Combining these equations yields

$$\dot{\sigma}(t) = \lambda\sigma(t)\,(1-\sigma(t))\,J(\sigma(t)). \tag{5}$$

The Youden index determines the evolution of the share of Better tenured scientists because it governs the gap between the tenure probabilities of Better and Worse scientists, which determines the difference between the growth rates of the populations of Better and Worse tenured scientists.

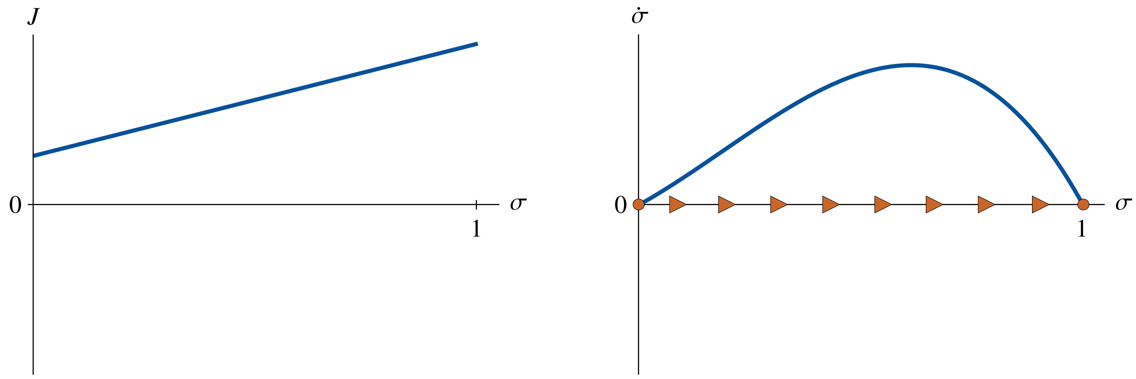
The dynamics of the population of scientists follow (5). Hence, we obtain the following results:

PROPOSITION 1: *Population dynamics depend on power. With high power ($1-\beta \geq \alpha + \epsilon/(1+\epsilon)$), the Better paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 1$), irrespective of initial conditions. With low power ($1-\beta < \alpha + \epsilon/(1+\epsilon)$), initial conditions matter: if the initial fraction of Better scientists is high ($\sigma(0) > \sigma^{*}$), the Better paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 1$); but if the initial fraction of Better scientists is low ($\sigma(0) < \sigma^{*}$), the Worse paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 0$).*

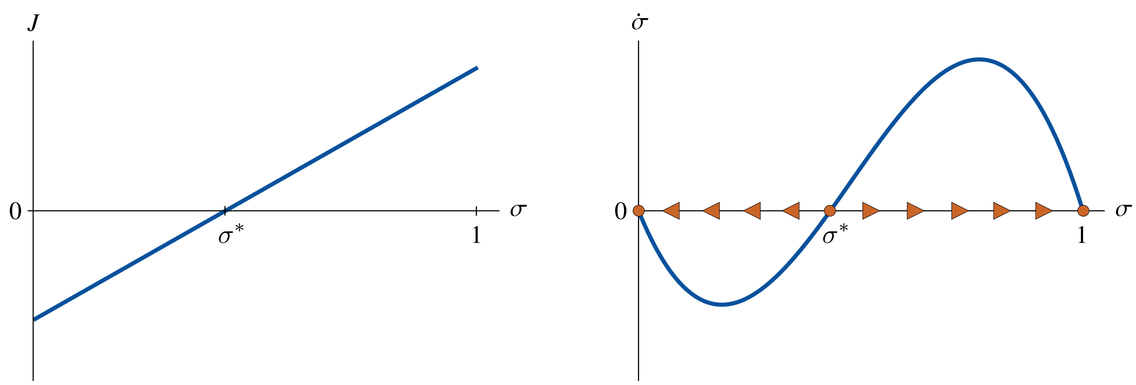
The proof is in appendix A but is illustrated in figure 1. For different levels of power, the Youden index has different properties, modifying the properties of (5)—the differential equation governing the dynamics of the population of scientists.

The proposition provides perspective on the conjecture by Friedman (1953, p. 11) that inferior paradigms will necessarily be abandoned, even if scientific tests have low power. If scientists are unbiased ($\epsilon = 0$), since the Better paradigm yields better research ($1-\alpha > \beta$), a science indeed always converges to the truth, irrespective of power. And, as Friedman anticipated, convergence is slower with lower power: without bias (5) becomes $\dot{\sigma} = (1-\beta-\alpha)\lambda\sigma(1-\sigma)$; when $1-\beta$ is lower, $\dot{\sigma}$ is lower for any σ, so convergence is slower.

But with homophilous bias ($\epsilon > 0$), branches of science with sufficiently low power ($1-\beta < \alpha + \epsilon/(1+\epsilon)$) are at risk of converging to inferior paradigms. So a lack of power does not just slow down convergence to superior paradigms: it changes the dynamics of science in such a way that superior paradigms may never be adopted.



A. High power: $1 - \beta \geq \alpha + \epsilon/(1 + \epsilon)$



B. Low power: $1 - \beta < \alpha + \epsilon/(1 + \epsilon)$

Figure 1. Population dynamics of a scientific field, depending on its power $(1 - \beta)$, significance (α), and homophilous bias (ϵ). The left-side graphs display (2), which relates the Youden index (J) to the share of Better scientists in the tenured population (σ). The right-side graphs are phase lines for (5), which describes how the share of Better scientists (σ) evolves over time. Panel A shows that when a science has high power, the Better paradigm necessarily prevails. Panel B shows that when a science has low power, the Worse paradigm may prevail; this happens when many scientists initially believe in the Worse paradigm $(\sigma(0) < \sigma^*)$.

In science, convergence is often used to validate paradigms. For instance, Friedman argues that the validity of the profit-maximization paradigm can be inferred from "its continued use and acceptance" (Friedman 1953, p. 23). In our model, this argument is incorrect: science may converge to an inferior paradigm, which becomes entrenched.

Adoption of an inferior paradigm occurs when power is low and the initial share of believers in the better paradigm is small ($\sigma(0) < \sigma^*$). Hence, because σ^* is higher when $1-\beta$ is lower, another cost of low power is to increase the size of the region where convergence to the inferior paradigm occurs. In fact, when the difference between power and significance falls to 0, irrespective of the amount of bias, the threshold σ^* rises to $1/2$. With such low power, then, even with low bias, convergence to the better paradigm occurs only if the majority of tenured scientists adopt that paradigm upon its inception.

Adoption of the Worse paradigm, when it occurs, is surprising because that paradigm describes the world less correctly than the Better paradigm, and Worse scientists do not have a larger bias than Better scientists. Why does the Worse paradigm prevail? Because once there are sufficiently many tenured Worse scientists, there is a high probability that one of the Worse scientists makes the tenure decision; as a result the tenure probability of Better scientists falls below that of Worse scientists. At this stage, the population of tenured Worse scientists grows faster than that of tenured Better scientists, and science converges to the Worse paradigm.

The proposition also points to a possible trigger of scientific revolutions: an increase in power. Assume that the fraction of Better scientists σ is converging to 0. To initiate a scientific revolution, the fraction of Better scientists needs to start converging to 1. This requires power to increase above

$$(1-\beta)^* = \alpha + \frac{\epsilon\,(1-2\sigma)}{1+\epsilon\,(1-2\sigma)}.$$

The threshold is computed such that the Youden index is 0 at σ. When power is above the threshold, the Youden index at σ turns positive, and science starts converging to the truth.

In appendix B, we introduce into the model additional mechanisms pulling science toward the truth or toward falsehood. The finding that Friedman's conjecture does not hold is reinforced: the conjecture does not hold even when tenure is decided by a committee of several scientists; even when Better scientists have stronger homophilous bias than Worse scientists; and even when Better scientists train more advisees than Worse scientists.

Four Historical Case Studies

The following case studies show that when a science has low power, superior paradigms may not be adopted; and then convergence to a superior paradigm only occurs if power increases. The case studies also show that power depends both on the data and methods available to test theories, and on the criteria used in promotion to the fellowship of established scientists.

The Nonadoption of Heliocentric Theory

We now know that the solar system is better described by the heliocentric than by the geocentric theory. Yet, the heliocentric theory had languished for almost 2,000 years before Copernicus; even after the publication of *De Revolutionibus* in 1543, in which Copernicus developed his heliocentric model of the solar system, there were "few converts for almost a century" (Kuhn 1996, p. 150). This does not mean that Copernicus had no immediate influence: Erasmus Reinhold based important new astronomical tables on the methods of *De Revolutionibus* only eight years after its publication. But contemporary astronomers, including Reinhold, did not believe in the moving earth (Kuhn 1957).

Adoption of the heliocentric theory only started with the discovery of high-power scientific tests of the geocentric and heliocentric theories. A first step toward acceptance of heliocentric theory came some 50 years after Copernicus' death, with observations of the elliptical orbits of the planets by Kepler, the first true Copernican after Copernicus' devoted pupil Rheticus (Repcheck 2007). But the real breakthrough came after 1609 with observations from Galileo's new high-resolution telescopes (Kuhn 1957). Prior to Galileo, when an observation did not quite fit with the Ptomelaic geocentric model, it was easily explained away by adding epicycles. With the power of the new telescopes such explanations became more difficult to countenance. It was particularly difficult to reconcile the Ptolemaic model with observations of the moons of Jupiter and the phases of Venus. After these observations, belief in the Copernican model spread rapidly.

The Nonadoption of Underconsumption Theory

Unlike the physical sciences, the social sciences lack the empirical evidence required to obtain high power. Even in economics, power seems low (Ioannidis, Stanley, and Doucouliagos 2017). According to our model, this lack of power puts the field at risk of being captured by false paradigms. An example from macroeconomics illustrates such risk: it shows a new, better paradigm that languished for almost half a century in the absence of high-power scientific tests to distinguish between the new paradigm and the old.

In 1887, Uriel Crocker, a Boston lawyer, published an article in the *Quarterly Journal of Economics* regarding the possibility of "an excess of [productive capacity] . . . beyond the amount required to meet all demands that are backed by the ability and the willingness to pay for the things demanded" (Crocker 1887, p. 362). Harvard professor Silas Macvane followed the article with a comment that concluded: "Demand for savings is the offer of labor for wages. In order that the supply of capital shall exceed the demand for it, there must be more capital offering for labor than the laborers are willing to receive! The mere statement of the case is sufficient to show its absurdity."

Undaunted, Crocker wrote a book, *The Cause of Hard Times*, in which he developed his underconsumption theory. But rather than becoming known as precursor to Keynes, Crocker was ignored. His distress is expressed in the last chapter: "in closing, it may be well to say that no professional economist has ever publicly recognized the validity of the theories and arguments set forth in this book" (Crocker 1895, p. 103). Among the economists who "published attempted refutations" or who "privately expressed to this author their complete dissent from his views" were luminaries of the profession, including J. Laurence Laughlin, Thorstein Veblen, and Frank Taussig. The Great Depression generated a powerful test of the old paradigm that supply creates its own demand; and, after Keynes' *General Theory*, economists no longer dismissed underconsumption theory as absurd.

The Persistence of Bloodletting

We have seen that power could be low because there are no high-power scientific tests to discriminate between true and false paradigms. But even if such tests do exist, they may play little role in promotion into the fellowship of senior scientists. The history of medicine illustrates this second possibility: high-power tests were available but played no significant role in promotion to the rank of practicing physician. Consequently, some procedures persisted decades after they had been shown to be harmful.

Bloodletting is a good example of such persistence. In the 1830s Pierre-Charles-Alexandre Louis, a practicing physician in Paris, took matched samples of pneumonia patients: one sample, with bloodletting in the first four days of the disease; the other sample, with bloodletting in days five to nine. Louis' results should at least have called for further testing, since he found a 76% higher fatality rate for those with early treatment (Rangachari 1997). That difference was difficult to explain if bloodletting was as beneficial to pneumonia patients as it was thought to be. When published in English (Louis 1836), Louis' findings were hailed in the *Journal of the American Medical Society* as "one of the most important medical works of the present century," being "the first formal exposition of the results of the only true method of investigation in regard to the therapeutic value of remedial agents" (Bartlett 1836, p. 102).

Yet Louis' use of statistical trials to determine the effects of bloodletting did not catch on. Neither did the later, more conclusive findings by Bennett (1865, 1866) have significant effect on practice. Bennett found no deaths among 105 patients whom he had treated for pneumonia without bloodletting at the Edinburgh Royal Infirmary; in contrast, when bloodletting had been standard treatment, more than one third of the pneumonia patients had died. The 1909 edition of Osler's influential textbook on *The Principals and Practice of Medicine* said that "local bloodletting by cupping or leeches is certainly advantageous in robust subjects" (Osler 1909, p. 782); such statements remained in posthumous editions, as late as 1942 (Thomas 2014).

Medical historian John Harley Warner culled doctors' letters and reports to explain why physicians were so averse to statistical methods. He concluded that physicians viewed themselves as professionals with clinical duties toward their patients; and that treatment would depend upon physicians' ability at observation, which was learned through their experience in practice. With this identity, it was considered denial of clinical duty to base judgment in individual cases on statistical samples of unknown patients in different locales and in different circumstances: "[Doctors] were not prepared to accept even in principle the proposition that they should discard existing therapeutic beliefs and practices, validated by both tradition and their own experience on account of somebody else's numbers" (Warner 1986, p. 201).

In sum, physicians and scientists had different norms regarding promotion. Kuhn's discussion of the Lavoisier revolution in chemistry illustrates the existence of a scientific norm: being a scientist entails accepting hypotheses that are confirmed by high-power tests. Lavoisier discovered a superior paradigm about combustion, which replaced an old paradigm that viewed combustion as occurring when flammable materials released their phlogiston. The difference between the two paradigms could be tested through the use of vacuums and precise weights; such tests had high power. Kuhn calls out Lavoisier's rival, Priestley, for being "unreasonable" because despite findings backed by high-power tests, Priestley resolutely continued his belief in the old phlogiston paradigm; Kuhn even engages in a rare demotion: he judges that Priestley "ceased to be a scientist" (Kuhn 1996, p. 159). In medicine, in contrast, there was no norm that a candidate's contribution to science should be evaluated with an eye on the results of high-power scientific tests; instead, physicians' criteria for promotion rested on a candidate's ability to carry out existing medical practice.

The Persistence of Radical Mastectomy

The history of radical mastectomy further illustrates the resistance of doctors to testing current procedures. Radical mastectomy was introduced in the United States in 1892 by Johns Hopkins' William Halsted. While the procedure was highly debilitating to its survivors, statistical evidence showed that it was not very effective. Indeed, matched statistics from the Cleveland Clinic published by Crile (1961) indicated that radical mastectomy yielded no improvement in mortality relative to simple mastectomy or lumpectomy, which were much less invasive. Despite the evidence, in 1968, 86% of surgical treatments for breast cancer were still by radical mastectomy (Lerner 2003, p. 132).

After the publication of Crile's findings, it took more than ten years before a significant-size randomized controlled trial was begun, against fierce opposition from the cancer-surgeon establishment (Lerner 2003). The opposition continued even when the trial was in progress. The breast-cancer surgeons, like Warner's 19th century physicians, based their resistance on their belief in the powers of clinical expertise. In an extreme expression of that opposition, the editor of the journal of the American Cancer Association said that use of randomized controlled trials to decide on procedures for individual patients was playing "scientific Russian roulette" with their lives (Lerner 2003, p. 115). When its results were published in 1981, 20 years after Crile's article, the randomized controlled trial bore out Crile's initial findings: no difference in mortality, but great difference in the condition of the survivors (Fisher et al. 1981).

In medicine, results from high-power tests played no role in promotion to the status of practicing physician. Instead, promotion depended on ability to execute current technique—especially in surgery. For example, trainees in breast-cancer surgery were admitted as practicing surgeons themselves based on their ability to carry out radical mastectomies; so that the promotion to elder of the profession had no regard for Crile's findings of little difference in mortality but much difference in patient welfare between radical and simple mastectomy.

Conclusion

This paper proposes a model regarding the adoption or nonadoption of superior scientific paradigms. The model captures parsimoniously—although perhaps a bit coarsely—Kuhn (1996)'s description of scientific revolutions. The model gives conditions under which inferior paradigms prevail when they are in contest with better ones. If scientific tests lack power, or are little used in determining admittance into the fellowship of established scientists, then the chances of getting trapped in an inferior paradigm are high. Lack of power does not just slow scientific progress; it may bring it to a halt.

In our model scientific progress is mediated by promotion to tenure, but the model could describe other aspects of the scientific process, such as hiring of junior faculty, award of grants and honors, and publication in scientific journals. Indeed, being hired by a university, receiving an award, or having an article accepted in a journal makes a scientist a prime candidate to be a reviewer for later applications or submissions—just as grantees of tenure become the judges of later tenure candidates. And the homophilous bias assumed in the tenure system has also been documented in hiring committees, award committees, and journals' peer-review system.

Kuhn left unanswered a question he deemed important: why has modern science been so successful? Our model suggests that two features of science have played an important role in its continuous progress. First, the physical sciences have made remarkable discoveries of high-power tests capable of distinguishing between true and false paradigms. Second, established scientists have been committed to admit into their ranks those whose work respects the findings of high-power tests, insofar as they are available.

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Appendix A. Proof of Proposition 1

Equation (5) is a logistic differential equation with threshold σ^*. This type of differential equation is well known: it is commonly used to model population dynamics—for instance, in ecology or in population genetics. Its properties are easy to derive.

We define a polynomial P by

$$P(\sigma) = \lambda\sigma\,(1-\sigma)\,J(\sigma),$$

where $\lambda > 0$ and $J(\sigma)$ is given by (2). Equation (5) can be written as $\dot{\sigma}(t) = P(\sigma(t))$, so the time path of $\sigma(t)$ is determined by the properties of P.

We first consider the case $\epsilon = 0$. Then $J(\sigma) = 1-\beta-\alpha > 0$, so the polynomial P has two roots, 0 and 1, and $P(\sigma) > 0$ for all $\sigma \in (0,1)$. Hence, differential equation (5) has two critical points: $\sigma = 0$ is a source, and $\sigma = 1$ is a sink. Consequently, for any initial condition $\sigma(0) \in (0,1)$, $\sigma(t)$ converges toward 1.

Next, we turn to the case $\epsilon > 0$ and

$$1-\beta \geq \alpha + \frac{\epsilon}{1+\epsilon}.$$

Then $J(\sigma) > 0$ for all $\sigma \in (0,1)$, so the polynomial P has two roots on $[0,1]$, 0 and 1, and $P(\sigma) > 0$ for all $\sigma \in (0,1)$. Hence, again, (5) has two critical points on $[0,1]$: $\sigma = 0$ is a source, and $\sigma = 1$ is a sink. Consequently, for any $\sigma(0) \in (0,1)$, $\sigma(t)$ converges toward 1.

Finally, we turn to the case $\epsilon > 0$ and

$$1-\beta < \alpha + \frac{\epsilon}{1+\epsilon}.$$

Then $J(\sigma) < 0$ for all $\sigma < \sigma^*$, $J(\sigma^*) = 0$, and $J(\sigma) > 0$ for all $\sigma > \sigma^*$, where $\sigma^* \in (0,1/2)$ is given by (3). Hence, the polynomial P has three roots, 0, σ^*, and 1, and for any $\sigma \in (0,1)$, $P(\sigma) > 0$ if $\sigma > \sigma^*$ and $P(\sigma) < 0$ if $\sigma < \sigma^*$. Accordingly, (5) has three critical points: $\sigma = 0$ is a sink, σ^* is a source, and $\sigma = 1$ is a sink. Thus, $\sigma(t)$ converges to 1 from any $\sigma(0) \in (\sigma^*, 1)$, and $\sigma(t)$ converges to 0 from any $\sigma(0) \in (0, \sigma^*)$.

Appendix B. Extensions of the Model

We analyze three extensions of our model: tenure committees are composed of several tenured scientists; Better and Worse scientists have different homophilous biases; and Better and Worse scientists train advisees at different rates. We find that the results generalize to all three cases.

Tenure Committees with Several Members

In universities, tenure cases are reviewed by tenure committees composed of several tenured faculty members. Furthermore, tenure committees request outside letters from a large number of experts in the field. Hence, we now assume that each untenured scientist is evaluated by a tenure committee composed of $2n - 1$ tenured scientists, with $n \geq 1$. The committee members are selected at random from the population of tenured scientists. The committee makes a decision by a majority vote: a candidate is granted tenure when at least n committee members vote favorably.

Let's first consider a Better tenure candidate. As in the baseline model, a member of the tenure committee is a Better scientist with probability σ and a Worse scientist with probability $1 - \sigma$. A Better committee member votes to deny tenure to the Better candidate with probability $(1 - \epsilon)\,\alpha$, whereas a Worse committee member votes to deny tenure to the Better candidate with probability $(1 + \epsilon)\,\alpha$. Accordingly, the probability that a committee member votes to deny tenure to the Better candidate is

$$A(\sigma) = \sigma\,(1 - \epsilon)\,\alpha + (1 - \sigma)\,(1 + \epsilon)\,\alpha = (1 + \epsilon)\,\alpha - 2\epsilon\alpha\sigma.$$

($A(\sigma)$ is exactly the probability that a Better candidate is denied tenure in the baseline model.) Hence, each committee member votes to deny tenure with probability $A(\sigma)$ and to grant tenure with probability $1 - A(\sigma)$. Since all committee members vote independently, the random variable counting the number of unfavorable tenure votes is a binomial random variable with parameters $A(\sigma)$ (probability that one vote is unfavorable) and $2n - 1$ (number of votes). The distribution function for this binomial random variable is

$$\mathcal{B}(x; A(\sigma), 2n - 1) = \sum_{i=0}^{x} \binom{2n-1}{i} A(\sigma)^i\ [1 - A(\sigma)]^{2n-1-i},$$

where x is an integer between 0 and $2n - 1$. Thus, the probability to grant tenure to a Better candidate, which is the probability that at most $n - 1$ committee members vote unfavorably, is

$$1 - \hat{\alpha}(\sigma) = \mathcal{B}(n - 1; A(\sigma), 2n - 1).$$

Following the same logic, the probability that a committee member votes to grant tenure to the Worse candidate is

$$B(\sigma) = \sigma\,(1-\epsilon)\,\beta + (1-\sigma)\,(1+\epsilon)\,\beta = (1+\epsilon)\,\beta - 2\epsilon\beta\sigma.$$

($B(\sigma)$ is exactly the probability that a Worse candidate is granted tenure in the baseline model.) Conversely, a committee member votes to deny tenure to the Worse candidate with probability $1 - B(\sigma)$. Once again, the random variable counting the number of unfavorable tenure votes is a binomial random variable with parameters $1 - B(\sigma)$ (probability that one vote is unfavorable) and $2n - 1$ (number of votes). The distribution function for this binomial random variable is

$$\mathcal{B}(x; 1 - B(\sigma), 2n-1) = \sum_{i=0}^{x} \binom{2n-1}{i} \, [1 - B(\sigma)]^i \; B(\sigma)^{2n-1-i}.$$

Thus, the probability to grant tenure to a Better candidate, which is the probability that at most $n - 1$ committee members vote unfavorably, is

$$\hat{\beta}(\sigma) = \mathcal{B}(n-1; 1 - B(\sigma), 2n-1).$$

Accordingly, with a tenure committee of $2n - 1$ members, population dynamics continue to be governed by the differential equation (5), where the Youden index is given by

$$J(\sigma) = 1 - \hat{\alpha}(\sigma) - \hat{\beta}(\sigma) = \mathcal{B}(n-1; A(\sigma), 2n-1) - \mathcal{B}(n-1; 1 - B(\sigma), 2n-1).$$

Although the Youden index admits a more complicated expression than in the baseline model, its main properties are unchanged, which will imply that population dynamics remain unchanged.

First, the Youden index is strictly increasing in $\sigma \in [0, 1]$. To see this, note that for any integers m and x such that $m \geq 1$ and $0 \leq x \leq m - 1$, a binomial distribution function $\mathcal{B}(x; p, m)$ is strictly decreasing in $p \in (0, 1)$. The intuition is simple: as the probability of success p of each of m independent Bernoulli trials increases, the probability that less than x of the Bernoulli trials are successful goes down—because each trial is more likely to be successful.[1] Second, note that $A(\sigma)$ and $B(\sigma)$ are strictly decreasing in σ. Combining these two results, we infer that $\mathcal{B}(n-1; A(\sigma), 2n-1)$ is strictly increasing in σ and $\mathcal{B}(n-1; 1 - B(\sigma), 2n-1)$ is strictly decreasing in σ. Accordingly, the Youden index $J(\sigma)$ is strictly increasing in $\sigma \in [0, 1]$.

[1]Formally, the binomial distribution function is given by

$$\mathcal{B}(x; p, m) = \sum_{i=0}^{x} \binom{m}{i} p^i \, (1-p)^{m-i}.$$

Second, when

$$1-\beta \geq \alpha + \frac{\epsilon}{1+\epsilon},$$

the Youden index is positive for all $\sigma \in (0,1)$. Indeed, under this condition,

$$1-B(0) = 1-(1+\epsilon)\,\beta \geq (1+\epsilon)\,\alpha = A(0),$$

which implies that

$$\mathcal{B}(n-1; A(0), 2n-1) \geq \mathcal{B}(n-1; 1-B(0), 2n-1)$$

and thus $J(0) \geq 0$. In addition, $J(\sigma)$ is strictly increasing in $\sigma \in [0,1]$, so $J(\sigma) > 0$ for all $\sigma \in (0,1)$.

Third, when

$$1-\beta < \alpha + \frac{\epsilon}{1+\epsilon},$$

the Youden index is negative for $\sigma < \sigma^*$, zero at $\sigma = \sigma^*$, and positive for $\sigma > \sigma^*$, where $\sigma^* \in (0, 1/2)$ is defined by (3). Indeed, by definition, $1-B(\sigma^*) = A(\sigma^*)$, which implies

$$\mathcal{B}(n-1; A(\sigma^*), 2n-1) = \mathcal{B}(n-1; 1-B(\sigma^*), 2n-1)$$

and thus $J(\sigma^*) = 0$. Moreover, $J(\sigma)$ is strictly increasing in $\sigma \in [0,1]$, so $J(\sigma) < 0$ for $\sigma < \sigma^*$, $J(\sigma) = 0$ at $\sigma = \sigma^*$, and $J(\sigma) > 0$ for $\sigma > \sigma^*$.

As the proof of proposition 1 solely relies on the three previous properties of the Youden index, we conclude that proposition 1 holds when tenure committees comprise $2n-1 \geq 1$ scientists, exactly as when tenure is decided by a single scientist. Hence, the dynamics of science

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Consider first the case $x < mp$. Taking the derivative of the distribution function with respect to p yields

$$\frac{\partial \mathcal{B}}{\partial p} = \sum_{i=0}^{x} \binom{m}{i} p^{i-1}\,(1-p)^{m-i-1}\,(i-mp).$$

Since $x < mp$, $i - mp < 0$ for all $i \leq x$, so $\partial \mathcal{B}/\partial p < 0$. Consider next the case $x \geq mp$. The binomial distribution function can be written as

$$\mathcal{B}(x; p, m) = 1 - \sum_{i=x+1}^{m} \binom{m}{i} p^{i}\,(1-p)^{m-i}.$$

Taking the derivative of the distribution function with respect to p yields

$$\frac{\partial \mathcal{B}}{\partial p} = -\sum_{i=x+1}^{m} \binom{m}{i} p^{i-1}\,(1-p)^{m-i-1}\,(i-mp).$$

Since $x \geq mp$, $i - mp > 0$ for all $i \geq x+1$, so once again we find $\partial \mathcal{B}/\partial p < 0$. We conclude that the binomial distribution function $\mathcal{B}(x; p, m)$ is strictly decreasing in p.

are independent of the size of tenure committees. In particular, the size of tenure committees does not influence the power threshold under which a science is at risk of converging to inferior paradigms (the threshold is $\alpha + \epsilon/(1+\epsilon)$, which is independent of n). And the size of tenure committees does not influence the range of initial conditions for which a science gravitates toward inferior paradigms (since σ^* is independent of n).

Overall, the analysis suggests that relying on a larger number of scientists to make tenure decisions (either by increasing the size of tenure committees in universities or by requesting more outside letters) will not change the nature of scientific progress: a scientific field will be neither more likely nor less likely to converge to the truth. The only aspect of scientific progress modified by the number of scientists involved in the tenure decision is the speed at which a field converges to the dominant paradigm, because the shape of the Youden index $J(\sigma)$ changes when n changes, which affects $\dot{\sigma}$ through (5). Of course, there might be other reasons for multiple-member tenure committees, such as the interchange of knowledge between the evaluators and fairness to the individual researcher. Finally, tenure committees narrowly defined are only one of the many evaluators who determine grant of tenure. Those evaluators also include those who have previously evaluated the candidate's prospects and record—previous hiring committees, grant committees, referees of publications, journal editors, and so on. Thus the property that our analysis remains substantially unchanged with compound evaluations is important to the robustness of our analysis.

Heterogeneous Homophilous Biases

Given that Better and Worse scientists have different views of the world, it is natural to allow them to have different homophilous biases. Hence we introduce two biases: $\epsilon^B \in [0,1]$ for Better tenured scientists, and $\epsilon^W \in [0,1]$ for Worse tenured scientists. Then, a Better evaluator denies tenure to a Better scientist with lowered probability $(1-\epsilon^B)\,\alpha$ and grants tenure to a Worse scientist with lowered probability $(1-\epsilon^B)\,\beta$. And, a Worse evaluator denies tenure to a Better scientist with increased probability $(1+\epsilon^W)\,\alpha$ and grants tenure to a Worse scientist with increased probability $(1+\epsilon^W)\,\beta$. (To ensure that all probabilities remain in $[0,1]$, we add the restrictions that $\alpha \leq 1/(1+\epsilon^W)$ and $\beta \leq 1/(1+\epsilon^W)$.)

As a result, Better scientists are denied tenure with probability

$$\hat{\alpha}(\sigma) = \sigma\left(1-\epsilon^B\right)\alpha + (1-\sigma)\left(1+\epsilon^W\right)\alpha = \left(1+\epsilon^W\right)\alpha - \left(\epsilon^B+\epsilon^W\right)\alpha\sigma.$$

Similarly, Worse scientists are granted tenure with probability

$$\hat{\beta}(\sigma) = \sigma\left(1-\epsilon^B\right)\beta + (1-\sigma)\left(1+\epsilon^W\right)\beta = \left(1+\epsilon^W\right)\beta - \left(\epsilon^B+\epsilon^W\right)\beta\sigma.$$

Given these tenure probabilities, the Youden index becomes

$$J(\sigma) = 1 - \hat{\alpha}(\sigma) - \hat{\beta}(\sigma) = 1 - \left(1+\epsilon^W\right)(\alpha+\beta) + \left(\epsilon^B+\epsilon^W\right)(\alpha+\beta)\sigma.$$

The Youden index is linearly increasing in $\sigma \in [0,1]$, from

$$J(0) = 1 - \left(1+\epsilon^W\right)(\alpha+\beta)$$

to

$$J(1) = 1 - \left(1-\epsilon^B\right)(\alpha+\beta).$$

Therefore, two cases arise. If

$$1-\beta \geq \alpha + \frac{\epsilon^W}{1+\epsilon^W},$$

the Youden index is positive for all $\sigma \in (0,1)$. And if

$$1-\beta < \alpha + \frac{\epsilon^W}{1+\epsilon^W},$$

the Youden index is negative for $\sigma < \sigma^*_\epsilon$, zero at $\sigma = \sigma^*_\epsilon$, and positive for $\sigma > \sigma^*_\epsilon$, where the threshold $\sigma^*_\epsilon \in (0, \epsilon^W/(\epsilon^B+\epsilon^W))$ is given by

$$\sigma^*_\epsilon = \frac{\epsilon^W}{\epsilon^B+\epsilon^W}\left[1 - \frac{1-(\alpha+\beta)}{\alpha+\beta}\cdot\frac{1}{\epsilon^W}\right].$$

Population dynamics continue to be governed by the differential equation (5). And since the Youden index retains similar properties as in the baseline model, we obtain a proposition similar to proposition 1:

PROPOSITION A1: *With heterogeneous homophilous biases, population dynamics depend on power. With high power ($1-\beta \geq \alpha + \epsilon^W/(1+\epsilon^W)$), the Better paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 1$), irrespective of initial conditions. With low power ($1-\beta < \alpha + \epsilon^W/(1+\epsilon^W)$), initial conditions matter: if the initial fraction of Better scientists is high ($\sigma(0) > \sigma^*_\epsilon$), the Better paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 1$); but if the initial fraction of Better scientists is low ($\sigma(0) < \sigma^*_\epsilon$), the Worse paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 0$).*

Here again, branches of science with homophilous bias ($\epsilon^W > 0$) and sufficiently low power

($1 - \beta < \alpha + \epsilon^W/(1 + \epsilon^W)$) are at risk of converging to inferior paradigms. Critically, the bias of Better scientists (ϵ^B) does not influence the possibility of convergence to the Worse paradigm; what matters is the bias of Worse scientists (ϵ^W). This is because for σ close to 0 there are almost no Better scientists, so their bias is irrelevant to determine whether the critical point 0 is a source or a sink, which in turn decides whether convergence to the Worse paradigm happens or not. Accordingly, even if Better scientists have stronger homophilous bias than Worse scientists, Friedman's conjecture is refuted: a lack of power does not just slow down convergence to the Better paradigm; it makes convergence to the Worse paradigm possible.

While the bias of Better scientists does not influence the possibility of convergence to the inferior paradigm, it does affect science dynamics in other ways. For instance, an increase in the bias of Better scientists decreases the threshold σ^*_ϵ and thus reduces the range of initial conditions for which science gravitates toward the Worse paradigm.

Heterogeneous Advising Rates

The race toward having a larger number of advisees suggests that advising is an important aspect of knowledge creation. Maybe some paradigms lend themselves to having more advising; for instance, because they are more applicable, or because they can be extended more easily. Hence we introduce different advising rates across paradigms: Better tenured scientists train advisees at rate $\lambda^B > 0$, and Worse tenured scientists train advisees at rate $\lambda^W > 0$.

The growth rates of the populations of Better and Worse tenured scientists are now given by

$$g^B(t) = \lambda^B \left(1 - \hat{\alpha}(\sigma(t))\right) - \delta$$
$$g^W(t) = \lambda^W \hat{\beta}(\sigma(t)) - \delta,$$

where, as in the baseline model,

$$\hat{\alpha}(\sigma) = (1 + \epsilon)\,\alpha - 2\epsilon\alpha\sigma$$
$$\hat{\beta}(\sigma) = (1 + \epsilon)\,\beta - 2\epsilon\beta\sigma.$$

We infer that

$$g^B(t) - g^W(t) = \lambda^B J_\lambda(\sigma(t)),$$

where $J_\lambda(\sigma(t))$ is a new Youden index, tailored to a situation with heterogeneous advising rates:

$$J_\lambda(\sigma) = 1 - \hat{\alpha}(\sigma) - \frac{\lambda^W}{\lambda^B}\hat{\beta}(\sigma) = 1 - (1 + \epsilon)\left(\alpha + \frac{\lambda^W}{\lambda^B}\beta\right) + 2\epsilon\left(\alpha + \frac{\lambda^W}{\lambda^B}\beta\right)\sigma.$$

The Youden index is linearly increasing in $\sigma \in [0,1]$, from

$$J_\lambda(0) = 1 - (1+\epsilon)\left(\alpha + \frac{\lambda^W}{\lambda^B}\beta\right)$$

to

$$J_\lambda(1) = 1 - (1-\epsilon)\left(\alpha + \frac{\lambda^W}{\lambda^B}\beta\right).$$

Here there are three cases to consider. First, if

$$1 - \frac{\lambda^W}{\lambda^B}\beta \geq \alpha + \frac{\epsilon}{1+\epsilon},$$

the Youden index is positive for all $\sigma \in (0,1)$. Second, if

$$1 - \frac{\lambda^W}{\lambda^B}\beta \leq \alpha - \frac{\epsilon}{1-\epsilon},$$

the Youden index is negative for all $\sigma \in (0,1)$. Third, if

$$1 - \frac{\lambda^W}{\lambda^B}\beta \in \left(\alpha - \frac{\epsilon}{1-\epsilon}, \alpha + \frac{\epsilon}{1+\epsilon}\right),$$

the Youden index changes sign on $(0,1)$: it is negative for $\sigma < \sigma^*_\lambda$, zero at $\sigma = \sigma^*_\lambda$, and positive for $\sigma > \sigma^*_\lambda$, where the threshold $\sigma^*_\lambda \in (0,1)$ is given by

$$\sigma^*_\lambda = \frac{1}{2}\left[1 - \frac{1 - \alpha - \left(\lambda^W/\lambda^B\right)\beta}{\alpha + \left(\lambda^W/\lambda^B\right)\beta} \cdot \frac{1}{\epsilon}\right].$$

The dynamics of $\sigma(t)$ continue to satisfy the differential equation (4). This equation can be rewritten as

$$\dot{\sigma}(t) = \lambda^B \sigma(t)\,[1-\sigma(t)]\,J_\lambda(\sigma(t)),$$

Exploiting the properties of the Youden index, we obtain the following proposition:

PROPOSITION A2: *With heterogeneous advising rates, population dynamics depend on power and advising rates. When*

$$1 - \frac{\lambda^W}{\lambda^B}\beta \geq \alpha + \frac{\epsilon}{1+\epsilon},$$

the Better paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t) = 1$)*, irrespective of initial conditions. When*

$$1 - \frac{\lambda^W}{\lambda^B}\beta \leq \alpha - \frac{\epsilon}{1-\epsilon},$$

the Worse paradigm eventually prevails ($\lim_{t\to\infty}\sigma(t)=0$*), irrespective of initial conditions. And when*

$$1-\frac{\lambda^W}{\lambda^B}\beta\in\left(\alpha-\frac{\epsilon}{1-\epsilon},\,\alpha+\frac{\epsilon}{1+\epsilon}\right),$$

initial conditions matter: if the initial fraction of Better scientists is high ($\sigma(0)>\sigma^*_\lambda$*), the Better paradigm eventually prevails (*$\lim_{t\to\infty}\sigma(t)=1$*); but if the initial fraction of Better scientists is low (*$\sigma(0)<\sigma^*_\lambda$*), the Worse paradigm eventually prevails (*$\lim_{t\to\infty}\sigma(t)=0$*).*

Here again, science is at risk of converging to inferior paradigms. This risk is present even if Better scientists train advisees at a higher rate than Worse scientists ($\lambda^B>\lambda^W$). Indeed, assume that the low-power condition of proposition 1 holds: $1-\beta<\alpha+\epsilon/(1+\epsilon)$. Then $1-(\lambda^W/\lambda^B)\beta<\alpha+\epsilon/(1+\epsilon)$ even when $\lambda^B>\lambda^W$, as long as λ^B/λ^W is not too large:

$$\frac{\lambda^B}{\lambda^W}\in\left(1,\frac{\beta}{1-\alpha-\epsilon/(1+\epsilon)}\right).$$

In that case, even though Better scientists train advisees at a higher rate than Worse advisees, Friedman's conjecture does not hold: there are initial conditions for which science never converges to the Better paradigm.

Friedman's conjecture holds for some advising rates, of course. For any level of power, if

$$\frac{\lambda^B}{\lambda^W}>\frac{\beta}{1-\alpha-\epsilon/(1+\epsilon)},$$

then $1-(\lambda^W/\lambda^B)\beta>\alpha+\epsilon/(1+\epsilon)$: science is guaranteed to converge to the truth. So if Better scientists train sufficiently more advisees than Worse scientists, the Worse paradigm will always be weeded out—even if power is low.

Finally, in the same way that advising by Better scientists helps science converge to the truth, advising by Worse scientists pushes science toward falsehood. And when Worse scientists advise sufficiently more students than Better scientists, scientific inquiry necessarily leads to falsehood. This happens when

$$\frac{\lambda^W}{\lambda^B}>\frac{1-\alpha+\epsilon/(1-\epsilon)}{\beta},$$

which ensures that $1-(\lambda^W/\lambda^B)\beta\leq\alpha-\epsilon/(1-\epsilon)$.