A Theory of Countercyclical Government Multiplier

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the government multiplier seems countercyclical

- US evidence:
  - Auerbach & Gorodnichenko [2012]
  - Candelon & Lieb [2013]
  - Fazzari, Morley, & Panovska [2015]

- international evidence:
  - Auerbach & Gorodnichenko [2013]
  - Jorda & Taylor [2016]
  - Holden & Sparrman [2016]
existing explanations

- the multiplier is large at the zero lower bound on the nominal interest rate
  - Eggertsson [2011]
  - Christiano, Eichenbaum, & Rebelo [2011]
  - Eggertsson & Krugman [2012]

- but evidence of countercyclical multipliers is obtained away from the zero lower bound
in this paper

- the government multiplier doubles when unemployment rises from 5% to 8%
  - irrespective of the zero lower bound
- mechanism based on the matching model of the labor market from Michaillat [2012]
  - unemployment = rationing + frictional
- multiplier $\equiv$ additional number of workers employed when 1 worker is hired in the public sector
public employment: main component of government consumption

- public employment = 63% of government consumption expenditures in the US, 1947–2011
  - even more if purchase of services (contractors) are included
- stimulus packages often raise public employment
  - example: Great Depression in the US
  - see Neumann, Fishback, & Kantor [2010]
the mechanism: crowding out

- public employment crowds out private employment
  - because government and firms compete for the same unemployed workers
- formally, an increase in public employment
  - raises labor market tightness
  - thus raises recruiting costs
  - which reduces private employment
the mechanism: bad times / good times

- **bad times**: labor demand is low so unemployment is high and competition for workers is weak
  - weak crowding out
- **good times**: labor demand is high so unemployment is low and competition for workers is strong
  - strong crowding out
- procyclical crowding out $\Rightarrow$ countercyclical multiplier
a matching model
with public employment
public employment

- the government employs $g_t$ workers
  - public employment is financed by an income tax
- public and private jobs are identical
  - same wage $w$
  - same job-separation rate $s$
- unemployed workers indiscriminately apply to public and private jobs
- public and private vacancies compete for the same unemployed workers
matching structure

$u$ unemployed workers

$\nu$ vacancies
matching structure

$u$ unemployed workers

$h$ newly employed workers

CRS matching function: $h = h(u, v)$

$h$ newly filled jobs

$\nu$ vacancies
matching structure

\[ \theta = \frac{v}{u} \]

job-finding probability:
\[ f(\theta) = \frac{h}{u} = m \cdot \theta^{1-\eta} \]

vacancy-filling probability:
\[ q(\theta) = \frac{h}{v} = m \cdot \theta^{-\eta} \]
worker flows: job creation and separation

1 - $u_t$ employed workers

$u_t$ unemployed workers
worker flows: job creation and separation

\[ f(\theta_t) \times u_t \]

\( n_t \) employed workers

\( u_t \) unemployed workers
worker flows: job creation and separation

\[ -s \times n_t \]

\( n_t \) employed workers

\( u_{t+1} \) unemployed workers
labor supply

- given $\theta$, the labor supply is workers’ employment rate when labor market flows are balanced

- balanced flows: $E \rightarrow U = U \rightarrow E$
  
  - $s \cdot n = f(\theta) \cdot u = f(\theta) \cdot [1 - n + s \cdot n]$

- labor supply:

  $$n^s(\theta) = \frac{f(\theta)}{s + (1 - s) \cdot f(\theta)}$$

- labor supply: equivalent to the Beveridge curve
representative firm

- hires \( l_t - (1 - s) \cdot l_{t-1} \) new workers by posting vacancies
  - cost per vacancy: \( r \cdot a \)
  - vacancy-filling probability: \( q(\theta_t) \)

- employs \( l_t \) workers paid \( w \)

- production function: \( y_t = a \cdot l_t^\alpha \)
  - \( a \): level of technology
  - \( \alpha \in (0, 1] \): marginal returns to labor
firm’s problem

Given wage and tightness \( \{w, \theta_t\} \), the firm chooses employment \( \{l_t\} \) to maximize discounted profits

\[
\sum_{t=0}^{+\infty} \beta^t \cdot \left[ a \cdot l_t^\alpha - w \cdot l_t - \frac{r \cdot a}{q(\theta_t)} \cdot \left[l_t - (1 - s) \cdot l_{t-1}\right]\right]
\]

- Production
- Wage bill
- Hiring cost
- New hires
private labor demand

- first-order condition with respect to \( l \) in steady state:

\[
\alpha \cdot \alpha \cdot l^{\alpha - 1} = w + \left[ 1 - \beta \cdot (1 - s) \right] \cdot \frac{r \cdot a}{q(\theta)}
\]

- marginal product of labor

- wage

- recruiting cost

- given \( \theta \) and \( w \), the private labor demand is firms’ desired employment rate in steady state:

\[
\ell_d(\theta, w) = \left[ \frac{1}{\alpha} \cdot \left\{ \frac{w}{a} + \left[ 1 - \beta \cdot (1 - s) \right] \cdot \frac{r}{q(\theta)} \right\} \right]^{\frac{-1}{1 - \alpha}}
\]
wage schedule

- there are mutual gains from matching
- many wage schedules are consistent with equilibrium
- we assume a simple wage schedule: $w = \omega \cdot a^\gamma$
  - $\gamma = 0$: fixed wage (unresponsive to $a$)
  - $\gamma = 1$: flexible wage (proportional to $a$)
  - $\gamma \in (0, 1)$: partially rigid wage (subproportional to $a$)
aggregate labor demand

- using the wage schedule, we rewrite the private labor demand as a function of $\theta$ and $a$:

\[
l^d(\theta, a) = \left[ \frac{1}{\alpha} \cdot \left\{ \omega \cdot a^{\gamma-1} + [1 - \beta \cdot (1 - s)] \cdot \frac{r}{q(\theta)} \right\} \right]^{\frac{-1}{1-\alpha}}
\]

- aggregate labor demand:

\[
n^d(\theta, a, g) = l^d(\theta, a) + g
\]
steady-state equilibrium

- tightness equalizes labor supply and demand:

\[ n^s(\theta) = n^d(\theta, a, g) \]

- recession: low technology \( a \)
- expansion: high technology \( a \)
- stimulus: high public employment \( g \)
- note: in matching models, the convergence to steady state is almost immediate [Hall 2005]
equilibrium diagram

\[ n^d(\theta, a, g) \quad \text{labour demand} \]

\[ n^s(\theta) \quad \text{labour supply} \]

\[ \theta \]

\[ \theta \]

\[ n \]

unemployment

\[ 1 \]
properties of the multiplier
definition of the multiplier

- the multiplier is $\lambda \equiv \partial n / \partial g$
  - additional number of workers employed when 1 worker is hired in the public sector
- another expression: $\lambda = 1 + \partial l / \partial g$
  - 1: mechanical effect of public employment
  - $\partial l / \partial g < 0$: crowding out of private employment by public employment
  - weaker crowding out $\Rightarrow$ larger multiplier
assumptions

- $\alpha < 1$: the production function has diminishing marginal returns to labor
  - in $(n, \theta)$ plane, $n^d(\theta, a, g)$ is downward-sloping
- $\gamma < 1$: the wage is partially rigid
  - in $(n, \theta)$ plane, $n^d(\theta, a, g)$ shifts inward when $a$ rises
- these are the assumptions from Michaillat [2012]
properties of the multiplier

under the assumptions that $\alpha < 1$ and $\gamma < 1$:

- the multiplier is $> 0$ but $< 1$
  - there is crowding out of private employment by public employment
  - but crowding out is less than one-for-one

- the multiplier is larger when $a$ is lower
  - higher unemployment $\Rightarrow$ larger multiplier
  - because crowding out is weaker
positive multiplier: mechanism

Labor market tightness

Supply

Demand: expansion

Employment n
positive multiplier: mechanism

Labor market tightness vs. Employment n

- Supply
- Demand: expansion
- Demand+stimulus

dg > 0
positive multiplier: mechanism
positive multiplier: mechanism

Supply
Demand: expansion
Demand+stimulus

Labor market tightness vs Employment n

dn > 0
dl < 0
dg > 0
countercyclical multiplier: mechanism

Labor market tightness

Supply
Demand: expansion
Demand+stimulus

dn > 0

dl < 0
dg > 0

Employment n

0.9
0.95
1

1

2

2.0
countercyclical multiplier: mechanism

Employment $n$
Labor market tightness

Supply
Demand: recession
Demand+stimulus

d$g > 0$
d$n > 0$
d$l < 0$

d$n > 0$
d$l < 0$

d$g > 0$
intuition for the mechanism

- when unemployment is high:
  - the government hires unemployed workers who would not have been hired otherwise
  - so public employment does not affect private employment much

- but when unemployment is low:
  - the government hires workers that would have been hired by the private sector otherwise
  - so public employment heavily crowds out private employment
what happens if $\alpha = 1$?

- $\alpha = 1$: linear production function
  - in $(n, \theta)$ plane, the labor demand is horizontal
- if $\alpha = 1$, the multiplier $= 0$
  - a change in $g$ does not change equilibrium $\theta$ so crowding out is one-for-one
what happens if $\gamma = 1$?

- $\gamma = 1$: flexible wage
  - the labor demand is independent of $a$
  - as with Nash bargaining
- if $\gamma = 1$, the multiplier is acyclical
  - unemployment and tightness are independent of $a$ so crowding out is independent of $a$
a New Keynesian model
standard features

- fluctuations arise from technology shocks

- representative large household
  - works for intermediate-good firms
  - consumes final good
  - saves using nominal bonds

- representative final-good firm
  - uses intermediate goods as input
  - sells output on perfectly competitive market
standard features

- intermediate-good firms
  - use labor as input
  - sell output on monopolistically competitive market to final-good firm
  - set price subject to a price-setting friction

- monetary policy
  - interest-rate rule (Taylor rule)
nonstandard features

- labor market with matching structure from Michaillat [2012]
  - instead of perfect/monopolistic competition
- quadratic price-adjustment cost [Rotemberg 1982]
  - instead of Calvo [1983] pricing
- government consumption is public employment
  - instead of purchase of goods
equilibrium: 9 endogenous variables

- exogenous variables:
  \[ \{a_t, g_t\}_{t=0}^{\infty} \]

- endogenous variables:
  \[ \{\theta_t, n_t, l_t, w_t, \Lambda_t, c_t, y_t, R_t, \pi_t\}_{t=0}^{\infty} \]
equilibrium: labor market

- equation # 1: wage schedule
  
  \[ w_t = \omega \cdot a_t^\gamma, \quad \gamma < 1 \]

- equation # 2: labor supply
  
  \[ n_t = (1 - s) \cdot n_{t-1} + f(\theta_t) \cdot [1 - (1 - s) \cdot n_{t-1}] \]

- equation # 3: public-employment policy
  
  \[ n_t = l_t + g_t \]
equilibrium: production

- equation # 4: production function

\[ y_t = a_t \cdot l_t^\alpha, \quad \alpha < 1 \]

- equation # 5: resource constraint

\[ y_t - \frac{r \cdot a_t}{q(\theta_t)} \cdot [n_t - (1 - s) \cdot n_{t-1}] = c_t \cdot \left[ 1 + \frac{\phi}{2} \cdot \pi_t^2 \right] \]
equilibrium: bond market

- **equation # 6: Euler equation**

  \[ 1 = \beta \cdot \mathbb{E}_t \left[ \frac{R_t}{1 + \pi_{t+1}} \cdot \frac{c_t}{c_{t+1}} \right] \]

- **equation # 7: Taylor rule**

  \[ R_t = \frac{1}{\beta} \cdot (1 + \pi_t)^{\mu_r} \cdot (1 - \mu_R) \cdot (\beta \cdot R_{t-1})^{\mu_R} \]
equilibrium: firms

- equation # 8: optimal pricing decision

\[ \pi_t \cdot (\pi_t + 1) = \frac{1}{\phi} \cdot \frac{y_t}{c_t} [\varepsilon \cdot \Lambda_t - (\varepsilon - 1)] + \beta \cdot \mathbb{E}_t [\pi_{t+1} \cdot (\pi_{t+1} + 1)] \]

- equation # 9: optimal employment decision

\[ \Lambda_t \cdot \alpha \cdot l_t^{\alpha - 1} = \frac{w_t}{a_t} + \frac{r}{q(\theta_t)} - \beta \cdot (1 - s) \cdot \mathbb{E}_t \left[ \frac{c_t}{c_{t+1}} \cdot \frac{a_{t+1}}{a_t} \cdot \frac{r}{q(\theta_{t+1})} \right] \]
steady state \((n, \theta)\) with zero inflation

- **Equation # 2**: labor supply

  \[
n^s(\theta) = \frac{f(\theta)}{s + (1 - s) \cdot f(\theta)}
  \]

- **Equation # 8**: \(\Lambda = (\varepsilon - 1)/\varepsilon\)

- **Equation # 1**: \(w = \omega \cdot a^\gamma\)

- **Equation # 9**: firms' labor demand

  \[
  \frac{\varepsilon - 1}{\varepsilon} \cdot \alpha \cdot \left[l^d(\theta, a) \right]^{\alpha - 1}\ = \omega \cdot a^{\gamma - 1} + (1 - \beta \cdot (1 - s)) \cdot \frac{r}{q(\theta)}
  \]

  isomorphic to steady state in matching model
simulations
simulation method

simulate nonlinear model under perfect foresight using shooting algorithm:

- **scenario #1**: public employment without stimulus
  - value of $g$: $\hat{g}_t = \bar{g}$
  - value of any $x$: $\hat{x}_t$
  - solid blue lines in graphs

- **scenario #2**: public employment with stimulus
  - value of $g$: $g^*_t > \bar{g}$
  - value of any $x$: $x^*_t$
  - dashed red lines in graphs
computation of the multiplier

- the instantaneous multiplier in a simulation is

\[
\frac{n_t^* - \hat{n}_t}{g_t^* - \hat{g}_t}
\]

- the cumulative multiplier of a simulation is

\[
\frac{\sum_{t=0}^{T} n_t^* - \hat{n}_t}{\sum_{t=0}^{T} g_t^* - \hat{g}_t}
\]

- cumulative multipliers are parametrized by the peak of the unemployment rate in the simulation
response to positive technology shock

![Graphs showing the response to positive technology shock](image)

- **Technology**: The graph shows a decrease over time, indicating a reduction in technology levels.
- **Public employment**: The graph indicates an increase over time, suggesting an upward trend in public employment.
- **Labor market tightness**: This graph also shows an increase, indicating tightening of the labor market.
- **Private employment**: Similar to public employment, this graph shows an increase over time.
- **Unemployment**: The graph shows a decrease, indicating a reduction in unemployment.
- **Gross domestic product (GDP)**: This graph depicts a decrease over time, reflecting a decline in GDP.
instantaneous multiplier after positive shock
response to negative technology shock

- **Technology**
- **Public employment**
- **Labor market tightness**
- **Private employment**
- **Unemployment**
- **Gross domestic product (GDP)**
instantaneous multiplier after negative shock
counter cyclical cumulative multiplier
countercyclical cumulative multiplier

![Graph showing the relationship between unemployment rate and cumulative multiplier. The graph demonstrates a linear increase in the cumulative multiplier as the unemployment rate increases from 5% to 8%. The cumulative multiplier values range from 0 to 0.5.](image)
conclusion
summary

- this paper proposes a New Keynesian model in which the government multiplier doubles when unemployment rises from 5% to 8%

- mechanism behind countercyclical multiplier:
  - multiplier = 1 – crowding out
  - and crowding out of private employment by public employment is much weaker when unemployment is higher
applications and extensions

- the same mechanism explains the procyclicality of the macroelasticity of unemployment with respect to unemployment insurance
  - see Landais, Michaillat, & Saez [2010]
- the same mechanism applies to the product market
  - see Michaillat & Saez [2015]
- the multiplier determines optimal stimulus spending
  - see Michaillat & Saez [2015]