An Economical Business-Cycle Model

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Abstract

In recent decades in the US, slack on the product and labor markets has fluctuated a lot over the business cycle while inflation has been very stable. Motivated by this observation, we develop a business-cycle model in which fluctuations in demand and supply lead to fluctuations in slack but not in inflation. Our model extends the money-in-the-utility-function model by introducing matching frictions and including real wealth in the utility function. Matching frictions allow us to represent slack and to consider a general equilibrium with constant inflation. Wealth in the utility function enriches the aggregate demand structure. We use the model to study the effects of various aggregate demand and supply shocks, and to analyze several stabilization policies. As a byproduct, the model provides a microfoundation for the IS-LM model and an explanation for permanent liquidity traps with positive inflation and unemployment. Last, we combine directed search with costly price adjustments to generate a Phillips curve that describes the joint dynamics of inflation and slack in the medium run.

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1. Introduction

In recent decades in the US, slack on the product and labor markets has fluctuated a lot over the business cycle, while inflation has been very stable. Figure 1 displays two measures of slack on the product market (the rate of idle capacity and the rate of idle labor), one measures of slack on the product market (the rate of unemployment), and the core inflation rate. The measures of slack are very countercyclical, whereas core inflation is very stable around 2%. The Great Recession is a good example of this pattern: from the beginning of 2008 to the middle of 2009, the rate of idle labor increased from 19% to 33%, the rate of idle capacity from 24% to 40%, the rate of unemployment increased from 5% to 10%, while the core inflation rate only fell from 2.1% to 1.2%.[1] Of course a possible explanation for the stability of inflation is that monetary policy is able to maintain inflation constant. But this seems implausible for two reasons. First, the mandate of monetary policy is to stabilize both slack and inflation, so it is unlikely that monetary policy focuses only on stabilizing inflation. Second, there is good empirical evidence that in the short run monetary policy does not have much influence on inflation: most empirical studies find that monetary policy barely contributes to short-run price movements.[2]

Motivated by these observations we develop a model of the business cycle in which fluctuations in demand and supply lead to fluctuations in slack but not in inflation. Our model offers a perspective on business cycles which differs from that of the standard New Keynesian model—in that model fluctuations in demand and supply lead to fluctuations in inflation but not in slack. We use our model to analyze several monetary and fiscal policies. We contrast the effects of these policies in and out of liquidity traps.

Our model has a simple structure since it only adds two elements to the well-known money-in-the-utility-function model of Sidrauski [1967]. The first element is matching frictions on the market where self-employed households sell labor services to other households.[3] In a matching

[1]The behavior of inflation over the past two decades led Hall [2011] to argue that inflation is exogenous for all practical purposes. In fact, inflation has become extremely hard to forecast after 1984. Stock and Watson [2008] find that it has become exceedingly difficult to improve systematically upon simple univariate forecasting models, such as the random walk model of Atkeson and Ohanian [2001].

[2]For instance, Christiano, Eichenbaum and Evans [1999] find that inflation responds only modestly and with long delays to monetary policy: in their vector autoregressions, the response of inflation to monetary policy is not statistically significant and it has a lag of two years.

[3]In modeling matching frictions we follow Michaillat and Saez [2015*a*]. Michaillat and Saez also provide a broad range of evidence suggesting that matching frictions are prevalent on the US labor and product markets.

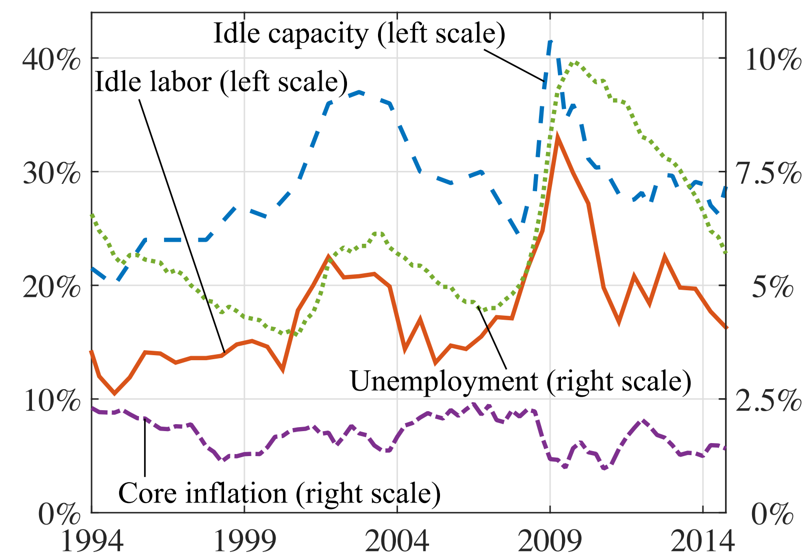


Figure 1: Slack and Inflation in the US, 1994–2014

Notes: The rate of idle capacity is one minus the rate of capacity utilization in the manufacturing sector measured by the Census Bureau from the Survey of Plant Capacity. The rate of idle labor is one minus the operating rate in the manufacturing sector measured by the Institute for Supply Management. The rate of unemployment is the civilian unemployment rate measured by the Bureau of Labor Statistics from the Current Population Survey. The rate of core inflation is the percent change from year ago of the personal consumption expenditures index (excluding food and energy) constructed by the Bureau of Economic Analysis as part of the National Income and Product Accounts. The rates of idle capacity and idle labor are quarterly series. The rates of unemployment and core inflation are quarterly averages of seasonally adjusted monthly series.

market households are unable to sell all their labor services, so slack emerges naturally. Furthermore, with a matching market there exists general equilibria with constant inflation; in these equilibria, market tightness adjusts to equalize aggregate supply and demand.[4]

The second element is the presence of real wealth in the utility function.[5] The motivation for this assumption is that people seem to care about real wealth not only as future consumption

[4]An alternative approach to representing constant inflation and slack is the general-disequilibrium approach of Barro and Grossman [1971]. The two approaches are fundamentally different. In our model the state of the economy is determined both by aggregate supply and by aggregate demand, even though inflation is constant. In contrast, in a general-disequilibrium model the state of the economy is determined by aggregate demand alone, independently of aggregate supply. This does not seem completely satisfactory because supply forces seem to matter in the short run; otherwise, we would not worry about changes in unemployment insurance, mismatch, labor force participation, and various subsidies and taxes.

[5]For others models in which consumption and wealth enter the utility function, see Kurz [1968], Zou [1995], Bakshi and Chen [1996], and Carroll [2000]. For a model in which consumption, money, and wealth enter the utility function, see Zou [1998]. These models have a different focus from ours. Kurz [1968] and Zou [1998] study long-term growth. Zou [1995] and Carroll [2000] study saving over the life cycle. Bakshi and Chen [1996] study portfolio choice and asset pricing. We study business cycles.

but for its own sake. People may value wealth because it is commonly used to rank people in societies and thus high wealth provides high social status.[6] People may also desire to accumulate wealth as an end in itself.[7] Several studies show that these considerations matter empirically. Weber [1930] shows the role of preferences for wealth in shaping macroeconomic outcomes.[8] Carroll [2000] shows that to explain some life-cycle saving patterns, wealth must enter the utility function.[9] Assuming wealth in the utility function provides a richer aggregate demand structure. This richer structure allows us to study a broader range of issues. For instance, a liquidity trap with positive inflation—and thus negative real interest rate—and high unemployment is a possible steady-state equilibrium of our model. Permanent liquidity traps exist because the consumption Euler equation is modified with wealth in the utility. Permanent liquidity traps raise challenging policy questions, and our model is useful to address some of them.

Our model is simple enough to inspect the mechanisms behind business cycles and analyze a broad range of stabilization policies. Indeed, the steady-state equilibrium is represented with an

[6] See for instance Smith [1776, Book II, Chapter III]. Smith observes that the desire to better one's condition is universal: "no man is so perfectly and completely satisfied with his situation as to be without any wish of alteration or improvement of any kind". Smith then observes that the only way to better one's condition in society is to accumulate wealth: "an augmentation of fortune is the means by which the greater part of men propose and wish to better their condition". He concludes that to better their condition people are forced to save because "the most likely way of augmenting their fortune is to save". The conclusion is that according to Smith, "the principle which prompts to save is the desire of bettering our condition", it is not the prospect of future consumption. In fact, Smith notes that because they want to better their condition above all, people tend to be "frugal".

[7] See for instance Keynes [1919, Chapter II]. Keynes observes that among the capitalist class of Europe before the First World War, "the duty of saving became nine-tenths of virtue and the growth of the cake the object of true religion", where the "cake" refers to the stock of capital in the economy. Keynes adds that while saving is a sign of virtue and thus a source of utility, consumption is viewed as a sin: "There grew round the non-consumption of the cake all those instincts of puritanism which in other ages has withdrawn itself from the world and has neglected the arts of production as well as those of enjoyment". And while in principle saving could be used for consumption during retirement or consumption by offsprings, Keynes argues that this did not happen: "Saving was for old age or for your children; but this was only in theory—the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you." Keynes [1931, Chapter V] also comes back to this thesis. Keynes observed that "the accumulation of wealth" was considered of "of high social importance" and that in the "last two hundred years", people had been accumulating money "as a possession", which he contrasted with accumulating money "as a means to the enjoyment and realities of life".

[8] The thesis of Weber [1930] is that the Protestant ethics favored frugality and asceticism and thus dignified the accumulation of wealth. As a result, the Protestant ethic catalyzed the development of capitalism in the Western world by promoting saving and capital accumulation. To support his thesis, Weber provides evidence that Protestant regions in Germany developed more rapidly than Catholic regions, and had more entrepreneurs.

[9] Carroll [2000] aims to explain the finding that the rich save much more than the poor (saving rate and lifetime income as well as marginal propensity to save and lifetime income are strongly positively correlated). Using simulations, he assesses the ability of several life-cycle models to explain the difference in saving between the rich and the poor. He concludes that the saving behavior of the richest households cannot be explained by models in which the only purpose of wealth accumulation is to finance future consumption, but can be explained by a model in which wealth is intrinsically desirable, or equivalently, wealth yields a flow of services such as power or social status.

IS and a LM curve depicted in a (consumption, interest rate) plane, and an AD and an AS curve depicted in a (consumption, market tightness) plane. The IS curve describes the trade-off between holding wealth and consumption. The LM curve describes the trade-off between holding money and consumption. The AD curve is obtained at the intersection of the IS and LM curves. The AS curve describes the supply of labor and the matching on the labor market. Furthermore, comparative statics completely describe the response of the equilibrium to unexpected shocks because there are no state variables and the system is a source so the equilibrium jumps from one steady state to another after such shocks.

Using the IS-LM-AD-AS representation, we analyze various aggregate demand and supply shocks and several monetary and fiscal policies. We find that a negative aggregate demand shock leads to lower output and lower tightness while a negative aggregate supply shock leads to lower output but higher tightness. After an aggregate shock, several policies are useful to stabilize the economy. A conventional monetary policy issuing money through open market operations can stabilize the economy in normal times but not in a liquidity trap—when the nominal interest rate falls to zero. In a liquidity trap, other policies can stabilize the economy: for instance, a helicopter drop of money, a wealth tax, or budget-balanced government purchases.

Although the approximation that inflation is constant seems useful and realistic to describe the short run, this approximation may be unsatisfactory to describe the medium run. To describe medium-run inflation fluctuations, we combine directed search as in Moen [1997] with costly price adjustments as in Rotemberg [1982]. We obtain a model in which inflation slowly responds to slack. Sellers decrease their prices when the economy is slack and increase them when the economy is tight, which generates inflation dynamics described by a Phillips curve. For instance, after a negative aggregate demand shock, slack increases and inflation decreases; the lower price level stimulates aggregate demand until the economy returns to the efficient steady state.

2. The Model

The model extends the money-in-the-utility-function model of Sidrauski [1967] by adding matching frictions on the market for labor services and wealth in the utility function. The economy consists of a measure 1 of identical households who hold money and bonds, produce labor services, and purchase labor services from other households for their own consumption.[10]

2.1. Money and Bonds

Households can issue or buy riskless nominal bonds. Bonds are traded on a perfectly competitive market. At time t, households hold $B(t)$ bonds, and the rate of return on bonds is the nominal interest rate $i(t)$.

A quantity $M(t)$ of money circulates at time t. Money is issued by the government through open market operations: the government buys bonds issued by households with money. At any time t, the quantity of bonds issued equals the quantity of money put in circulation: $-\dot{B}(t) = \dot{M}(t)$. Initially, $-B(0) = M(0)$. Therefore, at any time t,

$$-B(t) = M(t). \tag{1}$$

The representative household is net borrower: $B(t) \leq 0$. At time t, the revenue from seignorage is $S(t) = -B(t) \cdot i(t) = i(t) \cdot M(t)$. The government rebates this revenue lump sum to households. Without public spending or taxes, the government's budget is therefore balanced at any time.

Finally, money is the unit of account. The price level at time t is $p(t)$. The rate of inflation at time t is $\pi(t) = \dot{p}(t)/p(t)$. The quantity of real money in circulation at time t is $m(t) = M(t)/p(t)$.

2.2. The Market for Labor Services

We model the market for labor services as in Michaillat and Saez [2015*a*]. Households sell labor services on a market with matching frictions. Households would like to sell k units of services at any point in time. The capacity k of each household is exogenous. Households also consume labor services, but they cannot consume their own services, so they trade with other households. To buy labor services, households post $v(t)$ help-wanted advertisements at time t.

A matching function h with constant returns to scale gives the number of trades at time t: $y(t) = h(k, v(t))$. The matching function is twice differentiable, strictly increasing in both argu-

[10] We abstract from firms and assume that all production directly takes place within households. This is done to simplify the analysis. Michaillat and Saez [2015*a*] show how the model can be extended to include a labor market and a product market, distinct but formally symmetric, and firms hiring workers on the labor market and selling their production on the product market.

ments, and with diminishing marginal returns in both arguments. It also satisfies $0 \leq h(k, v(t)) \leq \min\{k, v(t)\}$.[11] In each trade, one unit of labor service is bought at price $p(t) > 0$.

The market tightness x is defined by $x(t) = v(t)/k$. With constant returns to scale in matching, the market tightness determines the probabilities to trade for sellers and buyers. At time t, one labor service is sold with probability $f(x(t)) = y(t)/k = h(1, x(t))$ and one help-wanted advertisement leads to a trade with probability $q(x(t)) = y(t)/v(t) = h(1/x(t), 1)$. We denote by $1-\eta$ and $-\eta$ the elasticities of f and q: $1-\eta \equiv x \cdot f'(x)/f(x) > 0$ and $\eta \equiv -x \cdot q'(x)/q(x) > 0$. We abstract from randomness at the household's level: at time t, a household sells $f(x(t)) \cdot k$ labor services and purchases $q(x(t)) \cdot v(t)$ labor services with certainty.

Households are unable to sell all their labor services since $f(x(t)) \leq 1$. Households are idle a fraction $1 - f(x(t))$ of the time. The rate of idleness can be interpreted as the unemployment rate in this economy of self-employed workers. Since h is strictly increasing in its two arguments, f is strictly increasing and q is strictly decreasing in x. This means that when the market tightness is lower, it is harder for households to sell their labor services but easier for them to buy labor services from others.

Posting help-wanted advertisements is costly. The flow cost of an advertisement is $\rho \geq 0$ units of labor services so that a total of $\rho \cdot v(t)$ recruiting services are spent at time t. These recruiting services represent the resources devoted to matching with an appropriate worker. Recruiting services are purchased like any other labor services. As output of labor services is used for consumption, denoted $c(t)$, and recruiting, we have $y(t) = c(t) + \rho \cdot v(t)$. Only labor services for consumption enter households' utility function; labor services for recruiting do not. Thus it is consumption and not output that matters for welfare. This definition of consumption is different from that in national accounts, where $y(t)$ would be called consumption, but defining consumption as output net of recruiting costs is common in the matching literature [for example, Gertler and Trigari, 2009].

The number of help-wanted advertisements is related to consumption by $q(x(t)) \cdot v(t) = y(t) = c(t) + \rho \cdot v(t)$. Therefore, the desired level of consumption determines the number of advertisements: $v(t) = c(t)/(q(x(t)) - \rho)$. Hence, consuming one unit of services requires to purchase $1 + \rho \cdot v(t)/c(t) = 1 + \tau(x(t))$ units of services where $\tau(x(t)) = \rho/(q(x(t)) - \rho)$. The function τ is positive and strictly increasing for all $x \in [0, x^m)$ where $x^m > 0$ satisfies $\rho = q(x^m)$. Furthermore,

[11] A matching function satisfying these properties is $h(k, v) = (k^{-\zeta} + v^{-\zeta})^{-1/\zeta}$ with $\zeta > 0$.

$\lim_{x \to x^m} \tau(x) = +\infty$. The elasticity of τ is $\eta \cdot (1 + \tau(x))$.

We characterize the efficient market tightness x^*, which maximizes consumption given the matching frictions. In equilibrium,

$$c(t) = \frac{y(t)}{1 + \tau(x(t))} = \frac{f(x(t))}{1 + \tau(x(t))} \cdot k. \tag{2}$$

Since $1/(1 + \tau(x)) = 1 - \rho/q(x)$ and $q(x) = f(x)/x$, we obtain

$$c(t) = (f(x(t)) - \rho \cdot x(t)) \cdot k. \tag{3}$$

This equation says that $\rho \cdot x(t) \cdot k = \rho \cdot v(t)$ units of services are dissipated in matching frictions. As established by Michaillat and Saez [2015*a*], the tightness that maximizes consumption , $x^* = \operatorname{argmax}\{(f(x) - \rho \cdot x) \cdot k\}$, is uniquely defined by $f'(x^*) = \rho$. An equivalent definition is $\tau(x^*) = (1 - \eta)/\eta$. This definition will be useful when we study the Phillips curve arising from costly price adjustment in Section 5. The efficient tightness is the tightness underlying the condition of Hosios [1990] for efficiency in a matching model.

The market can be in three regimes. The market is slack if a marginal increase in tightness increases consumption, tight if a marginal increase in tightness decreases consumption, and efficient if a marginal increase in tightness has no effect on consumption. Equivalently, the market is slack if $x(t) < x^*$, efficient if $x(t) = x^*$, and tight if $x(t) > x^*$. If tightness is efficient on average, then business cycles are a succession of slack and tight episodes.

Figure 2 summarizes the relation between market tightness and different quantities. Capacity, k, is a vertical line, independent of tightness. Output, $y = f(x) \cdot k$, is increasing in tightness as it is easier to sell services when tightness is high. Consumption, $c = f(x) \cdot k/(1 + \tau(x)) = (f(x) - \rho \cdot x) \cdot k$, first increases and then decreases in tightness. At the efficient tightness, the consumption curve is vertical. The difference between output and consumption are recruiting services, $\rho \cdot v = \rho \cdot k \cdot x$. The difference between capacity and output is idle capacity, $(1 - f(x)) \cdot k$.

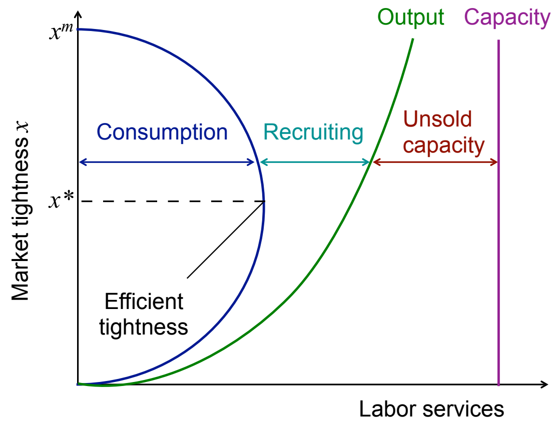


Figure 2: Capacity, Output, Unsold Capacity, and Consumption

2.3. Intertemporal Utility Maximization

Households spend part of their income on labor services and save part of it as money and bonds. The law of motion of the representative household's assets is

$$\dot{B}(t) + \dot{M}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) + i(t) \cdot B(t) + S(t).$$

Here, $M(t)$ are money balances, $B(t)$ are bond holdings, $p(t)$ is the price of services, $(1 + \tau(x(t))) \cdot c(t)$ is the quantity of services purchased, $f(x(t)) \cdot k$ is the quantity of services sold, and $S(t)$ is lump-sum transfer of seignorage revenue from the government. Let $A(t) = M(t) + B(t)$ denote nominal financial wealth at time t. The law of motion can be rewritten as

$$\dot{A}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot M(t) + i(t) \cdot A(t) + S(t).$$

Let $a(t) = A(t)/p(t)$ denote real financial wealth at time t and $s(t) = S(t)/p(t)$ real transfer of seignorage. Since $\dot{a}(t)/a(t) = \dot{A}(t)/A(t) - \pi(t)$, we have $\dot{a}(t) = \left(\dot{A}(t) - \pi(t) \cdot A(t)\right)/p(t)$, and the law of motion can be rewritten as

$$\dot{a}(t) = f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + r(t) \cdot a(t) + s(t), \tag{4}$$

where $r(t) \equiv i(t) - \pi(t)$ is the real interest rate at time t. This flow budget constraint is standard but for two differences arising from the presence of matching frictions on the labor market. First, income k is discounted by a factor $f(x(t)) \leq 1$ as only a fraction $f(x(t))$ of k is actually sold. Second, consumption $c(t)$ has a price wedge $1 + \tau(x(t)) \geq 1$ because resources are dissipated in recruiting: consuming one unit of services requires buying $1 + \tau(x(t))$ units of services.

Households experience utility from consuming labor services and holding real money balances and real wealth. Their instantaneous utility function is $u(c(t), m(t), a(t))$, where u is strictly increasing in its three arguments, strictly concave, and twice differentiable. The assumptions that real money balances and real wealth enter the utility function are critical to obtain a nondegenerate IS-LM system, and obtain permanent liquidity traps. The utility function of a household at time 0 is the discounted sum of instantaneous utilities

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot u(c(t), m(t), a(t))\, dt, \tag{5}$$

where $\delta > 0$ is the subjective discount rate. Throughout, $[x(t)]_{t=0}^{+\infty}$ denotes the continuous-time path of variable $x(t)$.

Definition 1. *The* representative household's problem *is to choose paths for consumption, real money balances, and real wealth $[c(t), m(t), a(t)]_{t=0}^{+\infty}$ to maximize (5) subject to (4), taking as given initial real wealth $a(0) = 0$ and the paths for market tightness, nominal interest rate, inflation, and seignorage $[x(t), i(t), \pi(t), s(t)]_{t=0}^{+\infty}$.*

Concretely, the model can be seen as the Sidrauski model with two additions. First, real wealth $a(t)$ enters the utility function. Second, matching frictions lower labor income by a factor $f(x(t))$ and increase the effective price of consumption by a factor $1 + \tau(x(t))$. Because $x(t)$ is taken as given by households, the model can be solved exactly as the original Sidrauski model. To solve the household's problem, we set up the current-value Hamiltonian:

$$\begin{aligned}\mathscr{H}(t, c(t), m(t), a(t)) =& u(c(t), m(t), a(t)) \\ &+ \lambda(t) \cdot [f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + r(t) \cdot a(t) + s(t)]\end{aligned}$$

with control variables $c(t)$ and $m(t)$, state variable $a(t)$, and current-value costate variable $\lambda(t)$.

Throughout we use subscripts to denote partial derivatives. The necessary conditions for an interior solution to this maximization problem are $\mathscr{H}_c(t, c(t), m(t), a(t)) = 0$, $\mathscr{H}_m(t, c(t), m(t), a(t)) = 0$, $\mathscr{H}_a(t, c(t), m(t), a(t)) = \delta \cdot \lambda(t) - \dot{\lambda}(t)$, and the transversality condition $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot \lambda(t) \cdot a(t) = 0$. Given that u is concave in (c, m, a) and that $\mathscr{H}$ is the sum of u and a linear function of (c, m, a), $\mathscr{H}$ is concave in (c, m, a) and these conditions are also sufficient.

These three conditions imply that

$$u_c(c(t), m(t), a(t)) = \lambda(t) \cdot (1 + \tau(x(t))) \tag{6}$$

$$u_m(c(t), m(t), a(t)) = \lambda(t) \cdot i(t) \tag{7}$$

$$u_a(c(t), m(t), a(t)) = (\delta - r(t)) \cdot \lambda(t) - \dot{\lambda}(t). \tag{8}$$

Equations (6) and (7) imply that the marginal utilities from consumption and real money balances satisfy

$$u_m(c(t), m(t), a(t)) = \frac{i(t)}{1 + \tau(x(t))} \cdot u_c(c(t), m(t), a(t)). \tag{9}$$

In steady state, this equation yields the LM curve. It represents a demand for money. The demand for real money is declining with $i(t)$ because $i(t)$ is the implicit price of holding money paying zero nominal interest instead of bonds paying a nominal interest rate $i(t)$.

Equations (6) and (8) imply that the marginal utilities from consumption and real wealth satisfy

$$(1 + \tau(x(t))) \cdot \frac{u_a(c(t), m(t), a(t))}{u_c(c(t), m(t), a(t))} + (r(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)}, \tag{10}$$

where $\dot{\lambda}(t)/\lambda(t)$ can be expressed as a function of $c(t)$, $m(t)$, $a(t)$, $x(t)$, and their derivatives using (6). This is the consumption Euler equation. In steady state, this equation yields the IS curve. It represents a demand for saving in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth.[12]

[12] If there are no matching costs ($\rho = 0$ and hence $\tau(x) = 0$) and if the utility only depends on consumption ($u_a = u_m = 0$), this Euler equation reduces to the standard continuous-time consumption Euler equation, $(r(t) - \delta) \cdot \varepsilon = \dot{c}(t)/c(t)$, where $\varepsilon \equiv -u'(c)/(c \cdot u''(c))$ is the intertemporal elasticity of substitution.

2.4. Equilibrium with Constant Inflation

We now define and characterize the equilibrium with constant inflation.

DEFINITION 2. *An* equilibrium with constant inflation π *consists of paths for market tightness, consumption, real money balances, money supply, real wealth, nominal interest rate, and price level,* $[x(t),\ c(t),\ m(t),\ M(t),\ a(t),\ i(t), p(t)]_{t=0}^{+\infty}$*, such that the following conditions hold: (i)* $[c(t), m(t), a(t)]_{t=0}^{+\infty}$ *solve the representative household's problem; (ii) monetary policy determines* $[M(t)]_{t=0}^{+\infty}$*; (iii) the money market clears; (iv) the bond market clears; (v) actual tightness on the market for services equals the tightness taken as given by households for their optimization problem; and (vi)* $[p(t)]_{t=0}^{+\infty}$ *is a continuous function of time t satisfying the differential equation* $\dot{p}(t) = \pi \cdot p(t)$ *with initial condition* $p(0) = 1$.

Conditions (i)–(v) are standard equilibrium conditions in a Walrasian economy. They impose that households optimize taking as given prices and trading probabilities, and that trading probabilities are realized.[13] These would be the equilibrium conditions in the Sidrauski model.

Conditions (i)–(v) generate six independent equations. Since the equilibrium consists of seven variables, the equilibrium definition is incomplete with these conditions only. This incompleteness arises from the presence of matching frictions on the market for labor services, which adds one aggregate variable—the market tightness. We therefore need a pricing mechanism to complete the equilibrium definition. It is common in the matching literature to use bargaining as a pricing mechanism. Here, we choose instead a pricing mechanism that generates constant inflation. Namely, we impose that the price process is exogenous and grows at constant inflation rate π. (The initial condition $p(0) = 1$ is a normalization.) The price process responds neither to slack nor to monetary policy. If $\pi = 0$, the price is constant over time. This criterion seems appropriate to describe the short run in the US because inflation has been very sluggish there since the 1990s.

PROPOSITION 1. *An equilibrium with constant inflation* π *consists of paths of market tightness, consumption, real money balances, money supply, real wealth, nominal interest rate, and price*

[13] In a Walrasian market, agents behave under the assumption that they can buy or sell any quantity at the posted price; that is, they take as given a trading probability of 1. The equilibrium requirement that supply equals demand ensure that agents can actually buy or sell the quantity they desire in equilibrium, ensuring that the trading probability of 1 is realized in equilibrium.

level, $[x(t), c(t), m(t), M(t), a(t), i(t), p(t)]_{t=0}^{+\infty}$*, that satisfy the following seven conditions: (i) equation* (9) *holds; (ii) equation* (10) *holds; (iii)* $[M(t)]_{t=0}^{+\infty}$ *is determined by monetary policy; (iv)* $m(t) = M(t)/p(t)$*; (v)* $a(t) = 0$*; (vi) equation* (3) *holds; and (vii)* $\dot{p}(t) = \pi \cdot p(t)$ *with* $p(0) = 1$.

The proposition offers a simple characterization of the equilibrium. The conditions listed in the proposition follow almost immediately from those in the definition. In particular, the condition that the bond market clears yields $a(t) = 0$, and the condition that actual tightness equals posted tightness yields (3).

3. Analysis of the Equilibrium

In this section we analyze the equilibrium with constant inflation. We represent the steady state of the equilibrium with an IS curve and a LM curve depicted in a (consumption, interest rate) plane, and an AD curve and an AS curve depicted in a (consumption, market tightness) plane. This graphical representation is useful to analyze the comparative static effects of shocks and policies in Section 4. We also study the transitional dynamics of the equilibrium.

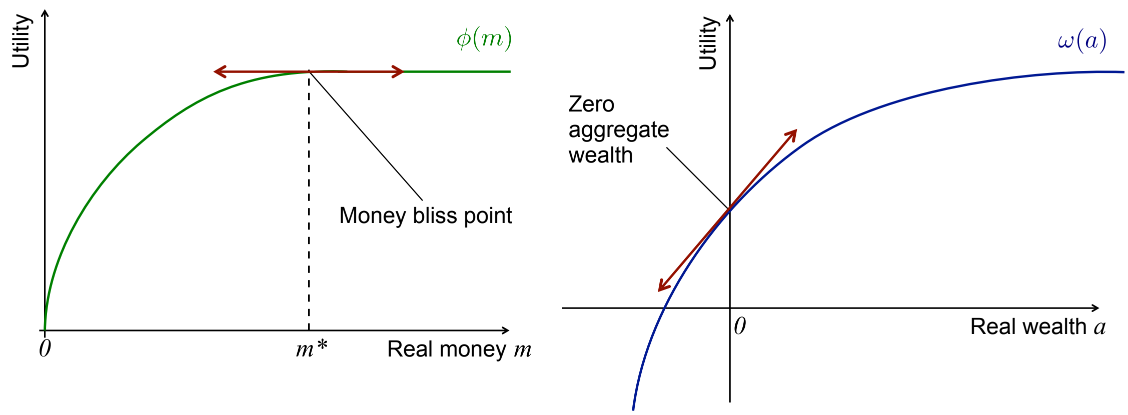
To obtain closed-form expressions for the curves, we assume that the utility function is separable in consumption, real money, and real wealth:

$$u(c, m, a) = \frac{\varepsilon}{\varepsilon - 1} \cdot \left(c^{\frac{\varepsilon-1}{\varepsilon}} - 1 \right) + \phi(m) + \omega(a). \tag{11}$$

The curvature of utility over consumption is measured by $\varepsilon \geq 1$. The function ϕ is strictly concave and strictly increasing on $[0, m^*]$ and constant on $[m^*, +\infty)$. The quantity $m^* \in (0, \infty)$ is a bliss point in real money balances; the bliss point is required to obtain liquidity traps. The function ω is strictly concave and strictly increasing on $(-\infty, +\infty)$. As wealth is zero in aggregate, the key parameter is the marginal utility of wealth at zero, $\omega'(0)$. We assume that $\omega'(0) \in (0, +\infty)$; a positive marginal utility of wealth is also required to obtain liquidity traps. The functions ϕ and ω are depicted in Figure 3.

3.1. IS Curve, LM Curve, AD Curve, and AS Curve

We define the IS, LM, AD, and AS curves that we use to represent the steady state.



A. Utility over real money

B. Utility over real wealth

Figure 3: The Utility Functions over Money and Wealth

Definition 3. *The* LM curve c^{LM} *is a function of nominal interest rate, market tightness, and real money balances defined by*

$$c^{LM}(i,x,m) = \left[\frac{i}{(1+\tau(x)) \cdot \phi'(m)}\right]^{\varepsilon}$$

for all $i \in [0,+\infty)$, all $x \in [0,x^m]$, and all $m \in [0,m^*)$. When real money balances are above the money bliss point ($m \geq m^*$), the LM curve determines a unique nominal interest rate: $i^{LM}(x,m) = 0$ for all $x \in [0,x^m]$ and all $m \in [m^*,+\infty)$. In this situation the economy is in a liquidity trap.

The LM curve is the amount of consumption that solves equation (9). The LM curve is defined separately for m below and above the bliss point because when m is above the bliss point, $\phi'(m) = 0$ so (9) is degenerate and imposes $i = 0$.

Definition 4. *The* IS curve c^{IS} *is a function of nominal interest rate, market tightness, and inflation defined by*

$$c^{IS}(i,x,\pi) = \left[\frac{\delta+\pi-i}{(1+\tau(x)) \cdot \omega'(0)}\right]^{\varepsilon}$$

for all $i \in [0,\delta+\pi]$, all $x \in [0,x^m]$, and all $\pi \in [-\delta,+\infty)$. If marginal utility of wealth is zero ($\omega'(0) = 0$), the IS curve determines a unique interest rate: $i^{IS}(x,\pi) = \pi + \delta$ for all $x \in [0,x^m]$ and all $\pi \in [-\delta,+\infty)$.

The IS curve is the amount of consumption that solves equation (10) when $\dot{\lambda}(t) = 0$. Although the IS curve is expressed with inflation and nominal interest rate, it only depends on the real interest rate, $r = i - \pi$. The IS curve is defined separately when the marginal utility of wealth is positive or zero because when the marginal utility of wealth is zero, (10) is degenerate and imposes $r = \delta$.

The properties of the IS and LM curves are illustrated in Figure 4.[14] First, Panel A shows that the LM curve is upward sloping in a (c,i) plane. This property follows the standard logic. Money does not pay interests; therefore, demand for real money is decreasing with i as a higher i increases the opportunity cost of holding money. Demand for real money is increasing in c as a higher c reduces the marginal utility of consumption, which makes real money more attractive relative to consumption. Given that real money balances are constant, an increase in i requires an increase in c to maintain equilibrium. Through the same logic, an increase in real money balances shifts the LM curve out, as illustrated in Panel C.

Second, Panel A shows that the IS curve is downward sloping in a (c,i) plane. The intuition is the following. For a given inflation, a higher i leads to a higher r and a higher marginal value of savings through bonds via the wealth effect $r \cdot \omega'(0)$, which makes holding wealth more attractive. Since wealth is zero in equilibrium, c must decline for households to be indifferent between saving and consumption. This logic also implies that an increase in inflation, which reduces r for a given i, shifts the IS curve out, as showed in Panel D. By the same logic, a decrease in the marginal utility of wealth shifts the IS curve out, as showed in Panel E. An increase in the discount rate has the same effect, as showed in Panel F.

Third, the IS and LM curves shift outward when market tightness decreases, as illustrated in Panel B. The logic is that a lower tightness reduces the effective price of labor services, $(1 + \tau(x)) \cdot p$, which makes consumption of labor services more desirable relative to holding bonds or money. However, the nominal interest rate defined by the intersection of the IS and LM curves does not depend on tightness: the IS and LM curves shift by commensurate amounts such that the equilibrium interest rate remains the same.

Fourth, the LM curve prevents the nominal interest rate from falling below zero because the marginal utility of money $\phi'(m)$ is nonnegative. If the nominal interest rate were negative, money would strictly dominate bonds. When real money is at or above the bliss point m^*, the LM curve becomes horizontal at $i = 0$, as illustrated in Panel A of Figure 5. Real money balances do not

[14]The linear IS and LM curves in Figure 4 correspond to the case with log utility over consumption ($\varepsilon = 1$).

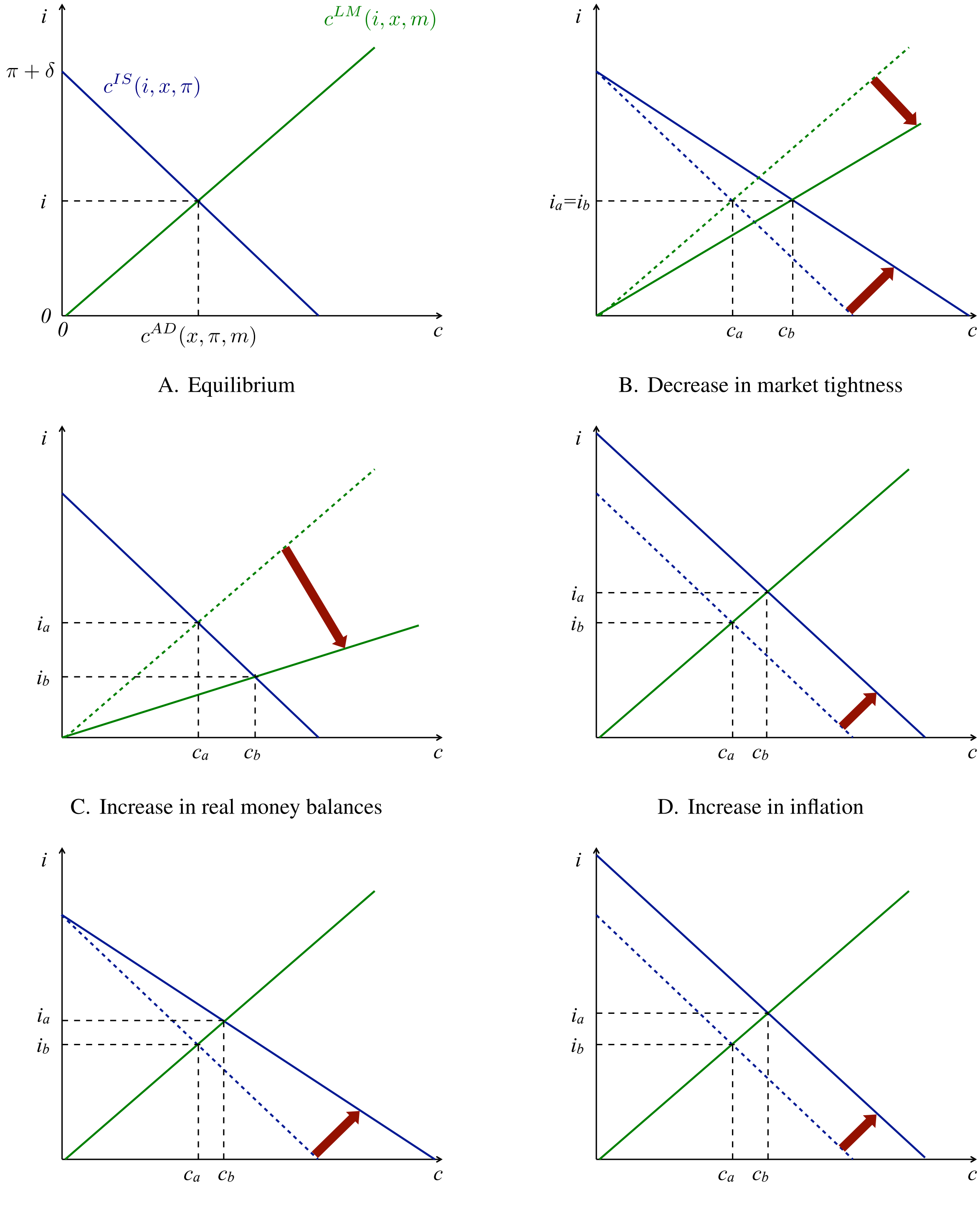


Figure 4: IS and LM Curves in (c,i) Plane

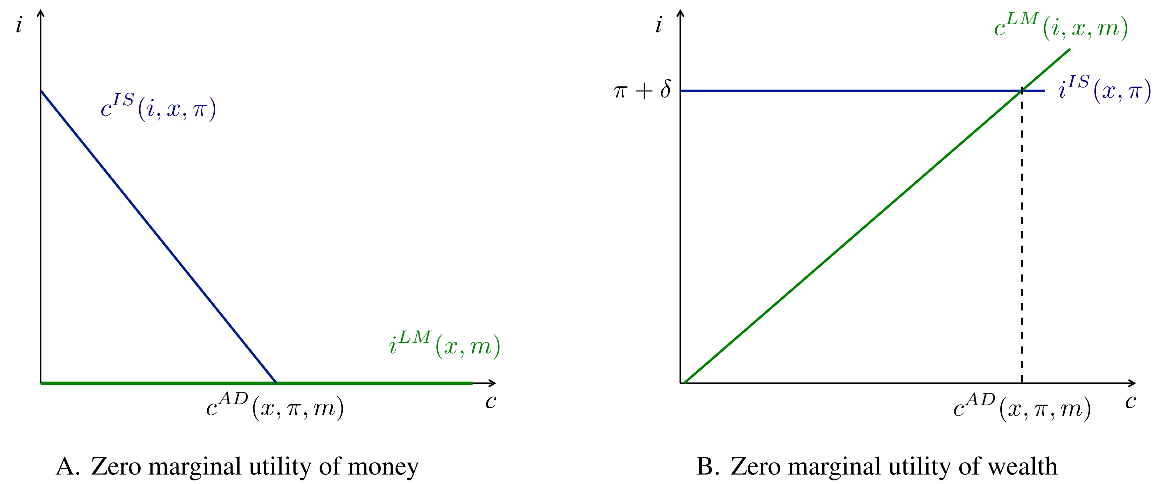


Figure 5: IS and LM Curves in Special Cases

affect the LM curve any more. This situation of liquidity trap has important implications to which we will come back.

Fifth, without utility of wealth, the IS curve becomes horizontal at $i = \delta + \pi$ as depicted in Panel B of Figure 5. The intuition is well known: steady-state consumption is constant so households hold bonds only if the return on bonds, $r = i - \pi$, equals the subjective discount rate, δ. With utility of wealth, $r < \delta$ as households also experience utility from wealth holding.

The equilibrium interest rate is given by the intersection of the IS and LM curves. The equality $c^{IS}(i,x,\pi) = c^{LM}(i,x,m)$ implies that the equilibrium nominal interest rate is

$$i = \frac{\phi'(m)}{\phi'(m) + \omega'(0)} \cdot (\delta + \pi). \tag{12}$$

At that interest rate households are indifferent between money and bonds. The equilibrium real interest rate is

$$r = \frac{\phi'(m)}{\phi'(m) + \omega'(0)} \cdot \delta - \frac{\omega'(0)}{\phi'(m) + \omega'(0)} \cdot \pi.$$

Next, we construct the AD curve by plugging (12) into the LM curve:

Definition 5. *The* AD curve c^{AD} *is a function of market tightness, inflation, and real money* *balances defined by*

$$c^{AD}(x,\pi,m) = \left[\frac{\delta+\pi}{(1+\tau(x))\cdot(\phi'(m)+\omega'(0))}\right]^{\varepsilon} \tag{13}$$

for all $x \in [0,x^m]$*, all* $\pi \in [-\delta,+\infty)$*, and all* $m \in [0,\infty)$.

The AD curve represents the consumption level obtained at the intersection of the IS and LM curves. The AD curve is downward sloping in a (c,x) plane, as illustrated in Panel A of Figure 8. The logic for this property is displayed in Panel B of Figure 4, where $c_a = c^{AD}(x_a,\pi,m)$, $c_b = c^{AD}(x_b,\pi,m)$ with $x_a > x_b$, and clearly $c_a < c_b$. In fact, all the properties of the AD curve follow from the mechanisms illustrated in Figure 4 and discussed above. For instance, the AD curve shifts out after an increase in the discount rate, an increase in the inflation rate, or a decrease in the marginal utility of wealth, as these changes shift the IS curve out. The AD curve also shifts out after an increase in real money balances, as this change shifts the LM curve out.

Last, we define the AS curve:

DEFINITION 6. *The* AS curve *is a function of market tightness defined by*

$$c^{AS}(x) = (f(x) - \rho\cdot x)\cdot k$$

for all $x \in [0,x^m]$.

The AS curve is the consumption level arising from the matching process on the labor market, plotted in Figure 2. The AS curve is showed in Panel A of Figure 8. An increase in capacity shifts the AS curve out.

3.2. Steady State

The steady state of the equilibrium with constant inflation is as follows:

PROPOSITION 2. *The steady state of the equilibrium with constant inflation* π *consists of market tightness, consumption, real money balances, level of money supply, growth rate of money supply, and nominal interest rate,* $(x,c,m,M(0),\dot{M}/M,i)$*, such that* $c^{LM}(i,x,m) = c^{IS}(i,x,\pi)$*,* $c^{AD}(x,\pi,m) = c^{AS}(x)$*,* $c = c^{AS}(x)$*,* $M(0)$ *is set by monetary policy,* $\dot{M}/M = \pi$*; and* $m = M(0)/p(0) = M(0)$.

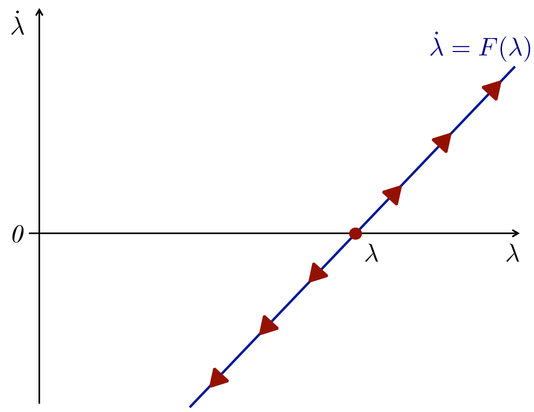


Figure 6: Phase Diagram of the Equilibrium with Constant Inflation

The steady state consists of 6 variables determined by 6 conditions. In steady state, the price grows at a constant, exogenous inflation rate π. The money supply, $M(t)$, must also grows at rate π but monetary policy does not control π. Hence, changing the growth rate of $M(t)$ is not within the scope of the analysis under constant inflation. Since the price level is unaffected by monetary policy, monetary policy controls real money balances by controlling the level of money supply.

When m is large enough ($m > m^*$), the steady state is in a liquidity trap, with a nominal interest rate at 0. This steady state is unique. Hence, the model easily accommodates permanent liquidity traps.[15] This is a desirable feature of the model since low-inflation economies such as the US seem prone to enter long liquidity traps after a large negative shock.[16]

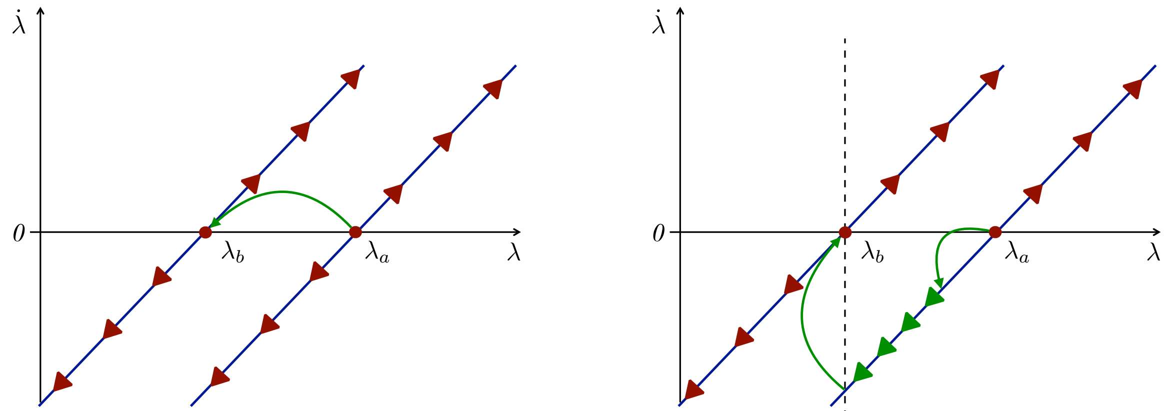
3.3. Transitional Dynamics

Here we describe the transitional dynamics toward the steady state. The dynamical system describing the equilibrium is characterized in Proposition 1. We focus here on one single endogenous variable: the costate variable $\lambda(t)$. All the variables can be recovered from $\lambda(t)$.

In equilibrium, wealth is zero so the law of motion for the costate variable from equation (8) is $\omega'(0) = (\delta + \pi - i(t)) \cdot \lambda(t) - \dot{\lambda}(t)$. Both money supply and price grow at a constant rate π

[15] The modern theory of liquidity trap was pioneered by Krugman [1998]. The macroeconomic model used by Krugman is much simpler than the standard New Keynesian model; a key simplification is that the price level is fixed during the liquidity trap. As discussed by Cochrane [2015], a number of difficulties arise when analyzing a liquidity trap with the standard New Keynesian model.

[16] The US economy entered a liquidity trap in December 2008 when the nominal interest rate set by the Federal Reserve reached its zero lower bound. It is still in this liquidity trap in March 2015.



A. Unexpected shock

B. Expected shock

Figure 7: Response of the Equilibrium with Constant Inflation to Unexpected and Expected Shocks

so real money balances are constant: $m(t) = M(0)/p(0) = m$. Hence, equation (7) implies that $i(t) \cdot \lambda(t) = \phi'(m)$, and the law of motion of the costate variable in equilibrium is

$$\dot{\lambda}(t) = (\delta + \pi) \cdot \lambda(t) - \omega'(0) - \phi'(m) \equiv F(\lambda(t)).$$

The steady-state value of the costate variable satisfies $F(\lambda) = 0$ so $\lambda = (\omega'(0) + \phi'(m))/(\delta + \pi)$. The nature of the dynamical system is given by the sign of $F'(\lambda)$. Since $F'(\lambda) = \delta + \pi > 0$, the system is a source. We represent the phase diagram for the system in Figure 6.

As there is no state variable, the system jumps from one steady state to the other in response to an unexpected shock—the transitional dynamics are immediate. This is illustrated in Panel A of Figure 7 where the equilibrium jumps from λ_a to λ_b at time t_0 when an unexpected shock occur. The values λ_a and λ_b are the steady-state values of λ for the parameters values before and after time t_0. Accordingly, comparative-statics analysis is sufficient to completely describe the behavior of the system after unexpected shocks.

The transitional dynamics are a bit different in response to an expected shock. This is illustrated in Panel B of Figure 7. An announcement is made at time t_0 that a shock changing the steady-state value of λ from λ_a to λ_b will occur at time t_1. A key property of the system is that absent new information, λ is a continuous variable of time so λ can only jump at time t_0 but not at time t_1. Assume that $\lambda_a > \lambda_b$. Then λ jumps down at time t_0. The amplitude of the jump is such that at

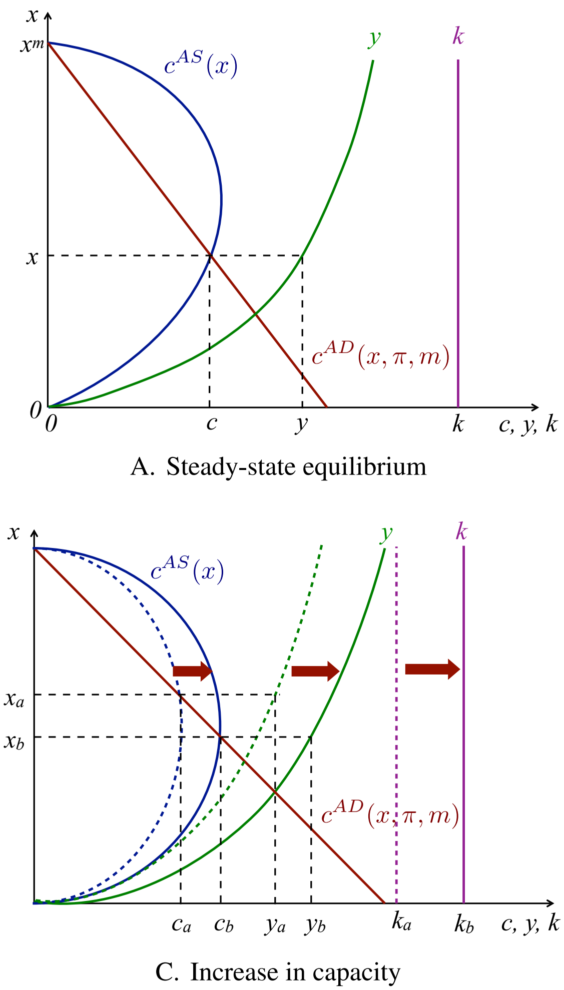


Figure 8: Steady-State Equilibrium and Aggregate Demand and Supply Shocks in a (c, x) Plane

time t_1, $\lambda = \lambda_b$. Between t_0 and t_1, λ falls because $\dot{\lambda} = F(\lambda) < 0$. We conclude that at time t_0, λ jumps down part of the way toward its steady-state value, and that it keeps on falling slowly toward its new steady-state value until the expected shock occurs. The implication is that even with expected shocks, comparative statics give the correct sign of the adjustments occurring when the announcement of the shock is made and in the long run.

4. Aggregate Shocks and Policies

In this section we use comparative statics to describe how the economy responds to aggregate demand and supply shocks and to various monetary and fiscal policies. As discussed in the previous section, comparative statics completely describe the response of the economy to an unexpected permanent shock because the equilibrium jumps from one steady state to another in response to such a shock. The comparative statics are summarized in Table 1 and illustrated in Figure 8.

4.1. Aggregate Demand Shocks

We first analyze aggregate demand shocks. We parameterize an increase in aggregate demand by an increase in the subjective discount rate or a decrease in the marginal utility of wealth. A positive aggregate demand shock shifts the IS curve out, as depicted in Panel E and F of Figure 4, and it therefore raises interest rates. Note that interest rates are independent of tightness, as illustrated in Panel B of Figure 4, so the general-equilibrium response of interest rates to the aggregate demand shock is the same as the partial-equilibrium response depicted in Panel E and F of Figure 4.

Since the IS curve shifts out, the AD curve also shifts out, as depicted in Panel B of Figure 8. Hence, the increase in aggregate demand leads to increases in market tightness and output. Since tightness is higher, the unemployment rate falls. Consumption increases if the labor market is slack and decreases if the labor market is tight. If the labor market is efficient, the aggregate demand shock has no first-order effect on consumption.

4.2. Aggregate Supply Shocks

Next we analyze aggregate supply shocks. We consider two types of shocks: a shock to the production capacity and a mismatch shock.

An increase in capacity is illustrated in Panel C of Figure 8. This increase shifts out the AS and output curves, while the AD curve is unchanged. Hence, consumption increases, tightness decreases, and the unemployment rate increases. We can show that output increases. Interest rates do not change.

Following Michaillat and Saez [2015*a*], we parameterize an increase in mismatch as a decrease in matching efficacy along with a commensurate decrease in matching costs: $h(k,v)$ becomes $\sigma \cdot h(k,v)$ and ρ becomes $\sigma \cdot \rho$ with $\sigma < 1$. Importantly, the efficient tightness and the function τ are not affected by mismatch. Panel D of Figure 8 illustrates an increase in mismatch. The AD curve does not change, but the AS and output curves shift inward. As a result, consumption decreases, tightness increases, and output decreases. We can show that the unemployment rate

Table 1: Comparative Statics: Aggregate Shocks and Policies with Constant Inflation

	Effect on:				
	Tightness	Consumption	Output	Unemployment rate	Interest rates
Increase in:	x	c	y	$1-f(x)$	i, r
Aggregate demand	+	+ / 0 / −	+	−	+
Capacity	−	+	+	+	0
Labor market mismatch	+	−	−	+	0
Money supply					
- out of liquidity trap	+	+ / 0 / −	+	−	−
- in liquidity trap	0	0	0	0	0
Helicopter money	+	+ / 0 / −	+	−	?
Wealth tax	+	+ / 0 / −	+	−	+
Government purchases	+	+ / 0 / −	+	−	0

Notes: An increase in aggregate demand results from an increase in the subjective discount rate or a decrease in the marginal utility of wealth. In the column on consumption, "$+/0/-$" indicates that consumption increases when the labor market is slack, does not change when the labor market is efficient, and decreases when the labor market is tight. In the column on interest rates, "?" indicates that the response of the interest rate can be positive or negative depending on the utility functions ω and ϕ. Given that inflation is constant, both nominal and real interest rate move in the same way. In the row on government purchase, consumption means total consumption—personal plus government consumption. Private consumption always falls when government consumption increases.

increases. Interest rates do not change.

The comparative statics are the same in a liquidity trap and away from it because the AD and AS curves retain the same properties in a trap. This property distinguishes our model from standard New Keynesian models, in which aggregate supply shocks have paradoxical effects in liquidity traps. In these models, a negative aggregate supply shock is contractionary in normal times but expansionary in a liquidity trap.[17]

4.3. Conventional Monetary Policy In and Out of a Liquidity Trap

The only lever that monetary policy chooses is the level of money supply, $M(0)$. A change in $M(0)$ leads to a change in real money balances. Monetary policy cannot change the growth rate of $M(t)$, which must satisfy the steady-state requirement that $\dot{M}(t)/M(t) = \pi$. We study the comparative

[17] See Eggertsson [2010, 2011] for a description of the paradoxical effects of aggregate supply shocks in a liquidity trap in New Keynesian models. Whether these paradoxical effects appear in the data is debated. For instance, using a variety of empirical tests, Wieland [2013] rejects the prediction that negative aggregate supply shocks are expansionary in a liquidity trap.

static effects of an increase in real money balances.

Away from a liquidity trap, an increase in real money balances shifts out the LM curve, as showed in Panel C of Figure 4, and hence shifts out the AD curve, as showed in Panel B of Figure 8. Higher money supply therefore leads to lower interest rates, higher tightness, lower unemployment rate, and higher output. The effect on consumption depends on the state of the labor market.

As long as the nominal interest rate is positive, monetary policy can control the AD curve and thus fully accommodate shocks. Suppose that the economy starts with tightness at its efficient level, which maximizes consumption, and that the government wants to use monetary policy to keep tightness at this level. A negative aggregate demand shock lowers tightness and requires an increase in real money balances, and conversely, a positive aggregate demand shock raises tightness and requires a decrease in real money balances. Here monetary policy absorbs aggregate demand shocks, thus preventing inefficient economic fluctuations. A positive aggregate supply shock, either an increase in capacity or a decrease in mismatch, lowers tightness and requires an increase in real money balances, and conversely, a negative aggregate supply shock raises tightness and requires a decrease in real money balances. Here monetary policy exacerbates the effect of aggregate supply shocks on output to achieve efficient economic fluctuations.

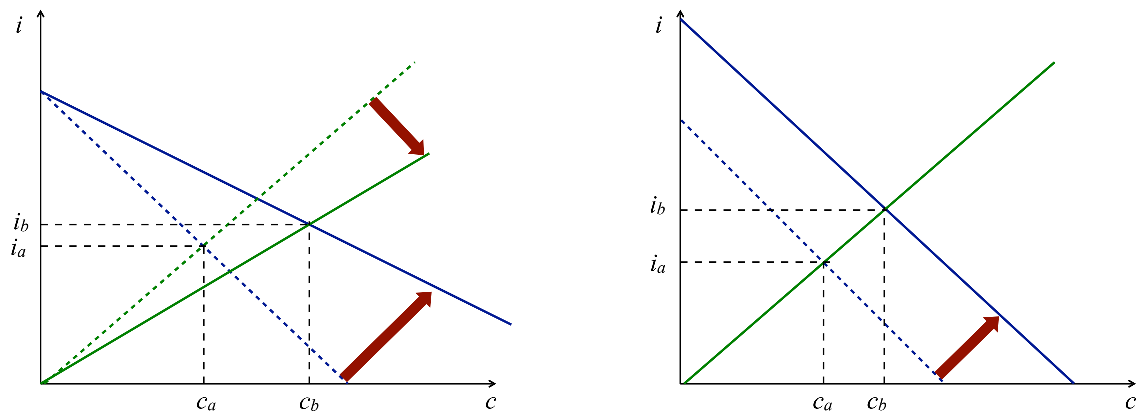
In a liquidity trap, monetary policy cannot accommodate shocks anymore because real money balances do not influence the LM curve and thus cannot control the AD curve. This situation is illustrated in Panel A of Figure 5. Monetary policy becomes ineffective. Of course, monetary policy could still be effective if it could change inflation. We know that higher inflation stimulates the IS curve and thus the AD curve, even in a liquidity trap, as showed in Panel D of Figure 4. But we assume here that monetary policy has no effect on inflation, consistent with the empirical evidence presented in Christiano, Eichenbaum and Evans [1999].

4.4. Helicopter Drop of Money

Conventional monetary policy is not effective in a liquidity trap. We now present other policies that remain effective in this situation.

We start by analyzing a helicopter drop of money.[18] Money comes from two sources: a quantity $M^b(t) = -B(t)$ of money is issued by buying bonds through open market operations, and a

[18] Friedman [1969] first discussed this policy.



A. Helicopter drop of money

B. Tax on wealth

Figure 9: Unconventional Policies in the IS-LM Diagram

quantity $M^h(t)$ of money is printed and given directly to households through a helicopter drop. Total money supply is $M(t) = M^b(t) + M^h(t)$. Real money balances are $m^b(t) = M^b(t)/p(t)$ and $m^h(t) = M^h(t)/p(t)$ and $m(t) = M(t)/p(t)$. Real wealth is no longer zero because helicopter money contributes to real wealth: real wealth is $a(t) = (B(t) + M^b(t) + M^h(t))/p(t) = m^h(t)$.

With helicopter money, our analysis carries over by adjusting the marginal utility of wealth from $\omega'(0)$ to $\omega'(m^h)$. The IS curve now depends on helicopter money:

$$c^{IS}(i, x, \pi, m^h) = \left[\frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(m^h)}\right]^{\varepsilon}.$$

Since the function ω is concave, an increase in helicopter money shifts the IS curve outward in a (c, i) plane, as showed in Panel A of Figure 9.[19] It also shifts the LM curve outward at it raises real money balances. The AD curve depends on both total and helicopter money:

$$c^{AD}(x, \pi, m, m^h) = \left[\frac{\delta + \pi}{(1 + \tau(x)) \cdot \left(\phi'(m) + \omega'(m^h)\right)}\right]^{\varepsilon}.$$

A helicopter drop of money shifts the AD curve out in a (c, x) plane, as in Panel B of Figure 8.

Although open-market money cannot stimulate the AD curve in a liquidity trap, helicopter money stimulates the AD curve even in a liquidity trap. Helicopter money is effective in a liquidity

[19]The efficacy of a helicopter drop of money requires concave utility of wealth. With linear utility of wealth, $\omega'(m^h)$ is constant and helicopter money does not shift the IS curve. In that case, a helicopter drop of money is ineffective.

trap because it stimulates both the LM and the IS curve, and the IS channel is immune to the liquidity trap. In contrast, open-market money only stimulates the LM curve, and the LM channel does not operate in a liquidity trap.

One drawback of a helicopter drop of money is that it is harder to reverse than open market operations. Effectively, reversing a helicopter drop of money requires to take away money from households with no compensation—taxing money held by households and destroying it.

4.5. Tax on Wealth

Another way to stimulate aggregate demand in a liquidity trap is to tax wealth at rate $\tau^a(t)$.[20] The wealth tax applies to the entire wealth, bond holdings plus money balances. The tax raises no revenue as the aggregate wealth is zero. But the tax changes the law of motion of the consumer's wealth and the consumption Euler equation. The law of motion becomes

$$\dot{a}(t) = f(x(t)) \cdot k - (1+\tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + (r(t) - \tau^a(t)) \cdot a(t) + s(t).$$

Therefore, the Euler equation becomes

$$(1+\tau(x(t))) \cdot \frac{u_a(c(t),m(t),a(t))}{u_c(c(t),m(t),a(t))} + (r(t) - \tau^a(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)},$$

and the IS curve admits a new expression:

$$c^{IS}(i,x,\pi,\tau^a) = \left[\frac{\delta + \tau^a + \pi - i}{(1+\tau(x)) \cdot \omega'(0)}\right]^{\varepsilon}.$$

An increase in the wealth tax shifts the IS curve outward in a (c,i) plane, as showed in Panel B of Figure 9. The LM curve remains the same. The AD curve is now a function of the wealth tax:

$$c^{AD}(x,\pi,m,\tau^a) = \left[\frac{\delta + \tau^a + \pi}{(1+\tau(x)) \cdot (\phi'(m) + \omega'(0))}\right]^{\varepsilon}.$$

[20] Correia et al. [2013] obtain a related result in a New Keynesian model. They show that replacing a labor income tax by a value-added tax can lift the economy of a liquidity trap. Since a value-added tax is equivalent to a tax on labor income and a one-time tax on existing wealth in a model without investment, suppressing a labor income tax and implementing a value-added tax instead amounts to implementing a one-time tax on wealth. This one-time tax on wealth is sufficient to stimulate the economy in a temporary liquidity trap. In our model with a permanent liquidity trap, a permanent wealth tax is needed.

An increase in the wealth tax shifts the AD curve outward in a (c,x) plane, as in Panel B of Figure 8. Since the wealth tax acts on the IS curve and not the LM curve, the wealth tax is effective in a liquidity trap. The intuition for the effectiveness of the tax is simple: taxing wealth makes holding wealth more costly and hence less desirable, hereby stimulating current consumption.

4.6. Government Purchases

The last policy that we consider is the purchase of $g(t)$ units of services by the government. We begin by assuming that government purchases are financed by a lump-sum tax $\tau(t)$. The government's budget constraint imposes that $p(t)\cdot g(t) = \tau(t)$. We assume that government purchases enters separately into households' utility function such that $g(t)$ does not affect the consumption and saving choices of the households. Accordingly, the IS and LM curves remain the same, and g only enters in the AD curve:

$$c^{AD}(x,\pi,m,g) = \left[\frac{\delta+\pi}{(1+\tau(x))\cdot(\phi'(m)+\omega'(0))}\right]^{\varepsilon} + \frac{g}{1+\tau(x)}. \tag{14}$$

We abuse notation and keep the labels c^{AD} and c^{AS} for the AD and AS curves, even though c is personal consumption whereas the AD and AS curves measure total consumption—personal plus government consumption. An increase in government purchases shifts the AD curve outward, as showed in Panel B of Figure 8.

Government purchases remain effective in a liquidity trap because they do not rely on the LM curve. However, government purchases are not especially effective in a liquidity trap. What matters for the effectiveness of government purchases are the slopes of the AD and AS curves. Following the logic described in Michaillat [2014], the government multiplier is higher when the economy is slack than when it is tight. But the multiplier is not particularly large in a liquidity trap since the slopes of the AD and AS curves are not particularly modified in a liquidity trap. To summarize, what matters for the size of the multiplier in our model is the amount of slack in the economy and not the liquidity trap. In that, our model sharply differs from the New Keynesian model, which predicts that government multipliers are much larger in a liquidity trap.[21]

[21] See for instance Christiano, Eichenbaum and Rebelo [2011], Eggertsson [2011], and Eggertsson and Krugman [2012].

An increase in government purchases leads to higher output but not always to higher total consumption. Following the usual logic, total consumption increases when the market is slack, decreases when the market is tight, and does not change when the market is efficient. Government consumption always crowds out personal consumption. Crowding out arises because an increase in government purchases shifts the AD curve outward and raises market tightness; therefore, it is more expensive for households to purchase goods: the effective price $(1+\tau(x))\cdot p$ increases. Households reduce consumption because of the increase in effective price. Crowding out is partial when the market is slack, one-for-one when the market is efficient, and more than one-for-one when the market is tight.

There is a simple interaction between fiscal and monetary policy. As long as monetary policy is able to maintain the market at efficiency, fiscal policy should follow public-finance considerations: the economy is always efficient so there is no reason to use government spending for stabilization purposes. If monetary policy cannot maintain the economy at efficiency, fiscal policy can play a role to stabilize the economy, in addition to public-finance considerations. This would happen for instance when the economy is in a liquidity trap and monetary policy cannot stimulate aggregate demand. Michaillat and Saez [2015*b*] formalize this discussion in their analysis of the optimal use of public-good spending for macroeconomic stabilization.

5. A Model with a Phillips Curve

In the previous sections inflation was constant. Although the approximation that inflation is constant seems useful and realistic to describe the short run, this approximation may be unsatisfactory to describe the medium run. In the medium run, a Phillips curve likely describes the joint dynamics of inflation and slack. In this section, we propose a version of the model that is more appropriate to describe the medium run. This version combines directed search as in Moen [1997] with costly price adjustment as in Rotemberg [1982]. In equilibrium, inflation dynamics are described by a Phillips curve.

To simplify the exposition, we specialize the utility (11) by setting $\varepsilon=1$ and $\phi(m)=\ln(m)$:

$$u(c,m,a)=\ln(c)+\ln(m)+\omega(a). \tag{15}$$

By using log utility over money, we set the money bliss point to infinity and ensure that the economy never enters a liquidity trap. Studying the properties of the equilibrium with Phillips curve in a liquidity trap is outside of the scope of this paper.[22] Finally, we assume that the money supply remains constant over time: $M(t) = M$ for all t. Unlike in a New Keynesian where monetary policy follows an interest-rate rule, monetary policy is completely passive here.

5.1. Intertemporal Profit Maximization

We begin by solving the representative seller's problem when buyers direct their search towards the most attractive markets but adjusting prices is costly to sellers. Buyers choose the market where they buy labor services based on the price, p, and tightness, x, in that market. What matters for buyers is the effective price they pay, $p \cdot (1 + \tau(x))$. When a seller sets a price, she takes into account the effect of her price on the tightness she faces, which in turn determines how much labor services she sells. The solution of the seller's problem yields a Phillips curve.

As in Moen [1997], we assume that sellers post their price $p(t)$ and that buyers arbitrage across sellers until they are indifferent across sellers. This means that search for labor services is not random but directed. For a given price $p(t)$, the tightness that a seller faces is given by

$$(1 + \tau(x(t))) \cdot p(t) = e(t) \tag{16}$$

where $e(t)$ is the effective price in the economy. The effective price is taken as given by buyers and sellers. This condition simply says that buyers are indifferent between all sellers. Sellers can choose high prices and get few buyers or low prices and get many buyers. If a seller chooses a price $p(t)$, her probability to sell therefore is

$$F(p(t)) \equiv f(x(t)) = f\left(\tau^{-1}\left(\frac{e(t)}{p(t)} - 1\right)\right).$$

A useful result is that the derivative of F is $F'(p) = -(1-\eta) \cdot f(x)/(\eta \cdot \tau(x) \cdot p)$. Absent any price-adjustment cost, sellers choose $p(t)$ to maximize $p(t) \cdot f(x(t))$ subject to (16); that is, they choose $x(t)$ to maximize $f(x(t))/(1+\tau(x(t))) = f(x(t)) - \rho \cdot x(t)$; thus, they set $f'(x(t)) = \rho$ and

[22] As the analysis of liquidity traps in New Keynesian models, this analysis raises difficult issues. See the discussions in Werning [2012] and Cochrane [2015].

$x(t) = x^*$ is efficient. This is the central efficiency result of Moen [1997].

We add price-adjustment costs to the directed search setting. We follow the price-adjustment specification of Rotemberg [1982]. Sellers incur a cost $(\dot{p}(t)/p(t))^2 \cdot \kappa(t)/2$ when they change their prices, where $\kappa(t) = \kappa \cdot p(t) \cdot y(t)$. This cost is quadratic in the growth rate of prices, $\dot{p}(t)/p(t)$, and scaled by the size of the economy $p(t) \cdot y(t)$ and a cost parameter κ. If $\kappa = 0$, prices adjust at no cost.

The representative seller takes $e(t)$, $\kappa(t)$, and $i(t)$ as given and chooses a price level $p(t)$, a price growth rate $\pi(t)$, and a tightness $x(t)$ to maximize the discounted sum of nominal profits

$$\int_0^{+\infty} e^{-I(t)} \cdot \left(p(t) \cdot f(x(t)) \cdot k - \frac{\kappa(t)}{2} \cdot \pi(t)^2 \right) dt, \tag{17}$$

subject to (16) and to the law of motion for the price level

$$\dot{p}(t) = \pi(t) \cdot p(t). \tag{18}$$

The seller's discount rate is $I(t) = \int_0^t i(s) ds$. To solve the seller's problem, we express $x(t)$ as a function of $p(t)$ using $f(x(t)) = F(p(t))$ and set up the current-value Hamiltonian

$$\mathscr{H}(t, \pi(t), p(t)) = p(t) \cdot F(p(t)) \cdot k - \frac{\kappa(t)}{2} \cdot \pi(t)^2 + \mu(t) \cdot \pi(t) \cdot p(t)$$

with control variable $\pi(t)$, state variable $p(t)$, and current-value costate variable $\mu(t)$. The necessary conditions for an interior solution to this maximization problem are $\mathscr{H}_\pi(t, \pi(t), p(t)) = 0$ and $\mathscr{H}_p(t, \pi(t), p(t)) = i(t) \cdot \mu(t) - \dot{\mu}(t)$, together with the appropriate transversality condition. The first condition implies that

$$\frac{\kappa(t)}{p(t)} \cdot \pi(t) = \mu(t). \tag{19}$$

Recall that $r(t) = i(t) - \pi(t)$ denotes the real interest rate. The second condition implies that

$$\dot{\mu}(t) = r(t) \cdot \mu(t) + \left(\frac{1-\eta}{\eta} \cdot \frac{1}{\tau(x(t))} - 1 \right) \cdot f(x(t)) \cdot k. \tag{20}$$

In a symmetric equilibrium, $\kappa(t)/p(t) = \kappa \cdot y(t) = \kappa \cdot f(x(t)) \cdot k$ so the first optimality condition simplifies to

$$\kappa \cdot f(x(t)) \cdot k \cdot \pi(t) = \mu(t). \tag{21}$$

As the elasticity of $f(x)$ is $1-\eta$, log-differentiating (21) with respect to time yields

$$\frac{\dot{\mu}(t)}{\mu(t)} = (1-\eta) \cdot \frac{\dot{x}(t)}{x(t)} + \frac{\dot{\pi}(t)}{\pi(t)}$$

Combining this equation with (20) yields

$$\dot{\pi}(t) = \left[r(t) - (1-\eta) \cdot \frac{\dot{x}(t)}{x(t)}\right] \cdot \pi(t) + \frac{1}{\kappa} \cdot \left(\frac{1-\eta}{\eta} \cdot \frac{1}{\tau(x(t))} - 1\right). \tag{22}$$

This differential equation describes sellers' optimal pricing; it underlies the Phillips curve.

5.2. The Dynamical System Describing the Equilibrium

Here we derive the dynamical system describing the equilibrium. The system is composed of three equations: a consumption Euler equation, a Phillips curve, and a law of motion for the marginal utility of money.

The consumption Euler equation describes the solution to the household's problem. It is given by (10), but it is convenient to rewrite it as a differential equation in x. Using (15), (10) becomes

$$\omega'(0) \cdot f(x(t)) \cdot k + r(t) - \delta = -\frac{\dot{\lambda}(t)}{\lambda(t)}.$$

Using the utility function (15) and the matching equation (2), the first-order condition (6) becomes $f(x(t)) \cdot k = 1/\lambda(t)$. Log-differentiating this equation with respect to time, we obtain

$$-\frac{\dot{\lambda}(t)}{\lambda(t)} = (1-\eta) \cdot \frac{\dot{x}(t)}{x(t)}$$

which yields the Euler equation

$$(1-\eta) \cdot \frac{\dot{x}(t)}{x(t)} = r(t) - \left(\delta - \omega'(0) \cdot f(x(t)) \cdot k\right). \tag{23}$$

We now turn to the Phillips curve. To ease notation, we denote the tightness gap as

$$G(x(t)) = 1 - \frac{1-\eta}{\eta} \cdot \frac{1}{\tau(x(t))}.$$

The function G increases in x, is positive if $x > x^*$, negative if $x < x^*$, and zero if $x = x^*$. It measures how far the market is from efficiency. Combining (22) with the Euler equation (23) to eliminate $x(\dot{t})$ yields the Phillips curve

$$\dot{\pi}(t) = \left(\delta - \omega'(0) \cdot f(x(t)) \cdot k\right) \cdot \pi(t) - \frac{1}{\kappa} \cdot G(x(t)) \tag{24}$$

The two differences with the usual Phillips curve in New Keynesian models is that the tightness gap, $G(x(t))$, replaces the usual output gap and the effective discount rate $\delta - \omega'(0) \cdot f(x) \cdot k$ replaces the usual discount rate δ. Using the Phillips curve, we can express inflation as the discounted sum of future tightness gaps: $\pi(t) = \int_t^\infty G(x(s)) \cdot f(x(s)) \cdot e^{R(t)-R(s)} ds / (\kappa \cdot f(x(t)))$ with $R(t) = \int_0^t r(s) ds$. This expression is obtained from integrating the differential equation (20) and using (21).

A last equation is required to describe the dynamics of the real interest rates, $r(t)$. This equation is based on the dynamics of real money balances. Let $\psi(t) = \phi'(M/p(t)) = p(t)/M$ denote the marginal utility of real money balances. Since M is fixed and $p(t)$ is a state variable, $\psi(t)$ is a state variable. As $\psi(t) = p(t)/M$, the law of motion of $\psi(t)$ is

$$\dot{\psi}(t) = \pi(t) \cdot \psi(t). \tag{25}$$

Using (7), we find that $i(t) = \psi(t) \cdot f(x(t)) \cdot k$ and $r(t) = i(t) - \pi(t) = f(x(t)) \cdot k \cdot \psi(t) - \pi(t)$. Hence we can rewrite the Euler equation as

$$\dot{x}(t) = \frac{x(t)}{1-\eta} \cdot \left[f(x(t)) \cdot k \cdot \left(\psi(t) + \omega'(0)\right) - \delta - \pi(t)\right]. \tag{26}$$

5.3. Properties of the Dynamical System

The dynamical system of (25), (24), and (26) describes the behavior over time of the vector $[\psi(t), \pi(t), x(t)]$ representing the equilibrium. It is a nonlinear system of differential equations. In this system, $x(t)$ and $\pi(t)$ are jump variables and $\psi(t)$ is a state variable. Proposition 3 determines the properties of the dynamical system:

PROPOSITION 3. *The vector $[\psi(t), \pi(t), x(t)]$ describing the equilibrium satisfies the dynamical system $\{(25), (24), (26)\}$. The dynamic system admits a unique steady state. This steady state has no inflation, efficient tightness, and an interest rate below the subjective discount rate: $\pi = 0$, $x = x^*$, and $i = \psi \cdot y^* = \delta - \omega'(0) \cdot y^*$, where $y^* = f(x^*) \cdot k$ is the efficient output level. Around the steady state, the dynamic system is a saddle, and the stable manifold is a line. Since the system has one state variable (ψ) and two jump variables (x and π), this property implies that the equilibrium is determinate. At the steady state, the stable manifold is tangent to the vector $\mathbf{z} = [\psi/\gamma_3, 1, (y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa]$, where $\gamma_3 = (\delta/2) \cdot \left[1 - \sqrt{1 + 4/(\kappa \cdot \delta^2 \cdot (1 - \eta))}\right] < 0$. The responses of the equilibrium to small shocks are determined by $\mathbf{z}$ and summarized in Table 2.*

Proof. In steady state, $\dot{\psi} = \dot{\pi} = \dot{x} = 0$. Since $\psi(t) > 0$, (25) implies that $\pi = 0$. There is no inflation in steady state, which is not surprising because there is no money growth. Since $\pi = 0$, (24) implies that $G(x) = 0$ and $x = x^*$. The market tightness is efficient in steady state. This means that prices always adjust in the long run to bring the economy to efficiency. The mechanism is that the price level determines real money balances and thus aggregate demand—this is the LM channel discussed in Section 4. This channel operates as long as the economy is not in a liquidity trap. Last, (26) with $\pi = 0$ implies that $(\psi + \omega'(0)) \cdot y^* = \delta$ with $y^* = f(x^*) \cdot k$. Thus $\psi = \delta/y^* - \omega'(0)$ and $i = \psi \cdot y = \delta - \omega'(0) \cdot y^*$.

To study the stability properties of the system around its steady state, we need to determine the eigenvalues of the Jacobian matrix $\mathbf{J}$ of the system evaluated at the steady state. Simple computations exploiting the fact that in steady state $\pi = 0$, $x = x^*$, $G(x^*) = 0$, $G'(x^*) = 1/x^*$, and

$(\psi + \omega'(0)) \cdot f(x^*) \cdot k - \delta = 0$, imply that

$$\mathbf{J} = \begin{bmatrix} 0 & \psi & 0 \\ 0 & \psi \cdot y^* & -\frac{1}{\kappa \cdot x^*} \\ \frac{y^* \cdot x^*}{1-\eta} & \frac{-x^*}{1-\eta} & \delta \end{bmatrix}.$$

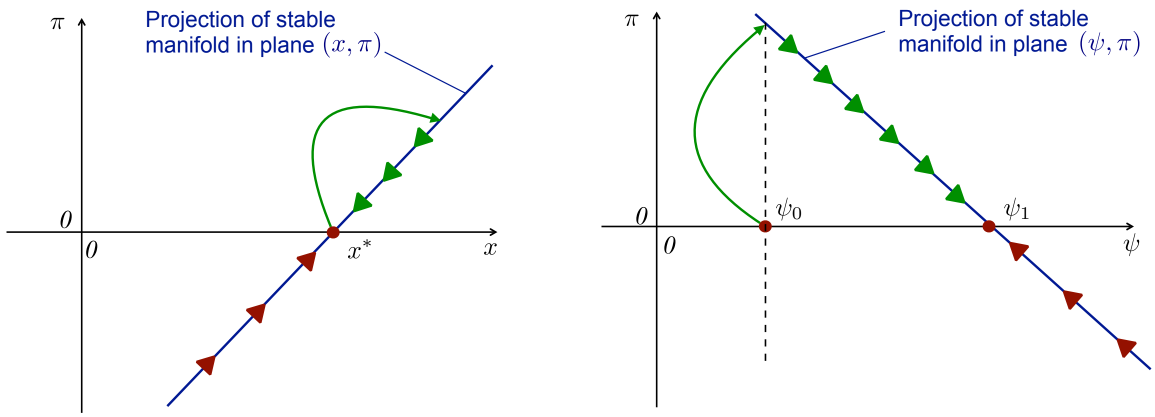
The characteristic polynomial of $\mathbf{J}$ is $(X - \psi \cdot y^*) \cdot \left[-X^2 + \delta \cdot X + 1/(\kappa \cdot (1-\eta))\right]$ so that $\mathbf{J}$ admits three real eigenvalues:

$$\begin{aligned} \gamma_1 &= \psi \cdot y^* > 0 \\ \gamma_2 &= \frac{\delta}{2} \cdot \left[1 + \sqrt{1 + \frac{4}{\kappa \cdot \delta^2 \cdot (1-\eta)}}\right] > 0 \\ \gamma_3 &= \frac{\delta}{2} \cdot \left[1 - \sqrt{1 + \frac{4}{\kappa \cdot \delta^2 \cdot (1-\eta)}}\right] < 0. \end{aligned}$$

Therefore, the system is a saddle path around the steady state, and the stable manifold is a line. Since the system has one state variable (ψ) and two jump variables (x and π), this property implies that the system does not suffer from dynamic indeterminacy. Suppose that the economy is at its steady state. In response to an unexpected and permanent shock at $t = 0$, both x and π jump to the intersection of the new stable line and the plane $\{\psi = \psi_0\}$, where ψ_0 denotes the old steady-state value of ψ. The economy remains on the plane $\{\psi = \psi_0\}$, orthogonal to the ψ axis, right after the shock because the state variable ψ cannot jump. This intersection is unique so the response of the system to the shock is determinate.

Finally, we compute the eigenvector $\mathbf{z}$ associated with the negative eigenvalue, γ_3. The stable line is tangent to $\mathbf{z}$ at the new steady state. Hence, this vector allows us to describe qualitatively the response of the equilibrium to aggregate demand and supply shocks, and monetary policy. The eigenvector is defined by $\mathbf{Jz} = \gamma_3 \mathbf{z}$. Simple calculation shows that this eigenvector is $\mathbf{z} = [\psi/\gamma_3, 1, (y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa]$. Using this vector, we obtain the responses to unexpected and permanent shocks described in Table 2. We now justify these responses.

We begin with two preliminary observations. First, irrespective of the shock, π and x always keep the same values in steady state, at $\pi = 0$ and $x = x^*$. The steady states are therefore all



A. Projection in the (x,π) plane B. Projection in the (ψ,π) plane

Figure 10: Response of the Equilibrium with Phillips Curve to an Unexpected Shock

aligned along a line $\{\pi = 0, x = x^*\}$, parallel to the ψ axis. Second, as $\gamma_3 < 0$, the coordinates of the eigenvector $\mathbf{z}$ along dimensions π and x are both positive—the coordinates are $1 > 0$ and $(y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa > 0$. This property is depicted in Panel A of Figure 10. Hence, x and π always move together in response to shocks—either they both increase, or they both decrease. In response to unexpected shocks, our Phillips curve therefore predicts a positive correlation between market tightness and inflation, or a negative correlation between unemployment and inflation, in the spirit of the traditional Phillips curve.

Next, consider a positive aggregate demand shock. For concreteness, assume that the marginal utility for wealth, $\omega'(0)$, decreases. In steady state, the marginal utility for money satisfies $\psi = \delta/y^* - \omega'(0)$ so a lower $\omega'(0)$ implies a higher ψ. Since the coordinate of the eigenvector $\mathbf{z}$ along dimension ψ is $\psi/\gamma_3 < 0$, π and x necessarily jump up on impact. This jump is depicted in Panel B of Figure 10. The responses of y and i follow because $y = f(x) \cdot k$ and $i = \psi \cdot y$. The mechanism is that after the shock, prices cannot adjust immediately so the economy becomes tight; unemployment is lower and output is higher than efficient. As sellers face a tight market, they increase prices—inflation is positive. As prices rise, real money balances decrease and ψ increases. This adjustment continues until the new steady state is reached. A monetary policy shock defined as a change in money supply has the same effects on all variables except i.

Last, a positive aggregate supply shock has exactly the same effect on tightness and inflation as a negative aggregate demand shock. For concreteness, assume that capacity, k, increases. In steady

Table 2: Dynamic Response of the Equilibrium with Phillips Curve to Shocks

	Short-run / long-run effect on:				
	Tightness	Inflation	Price	Output	Interest rate
Increase in:	x	π	p	y	i
Aggregate demand	$+$ / 0	$+$ / 0	0 / $+$	$+$ / 0	$+$ / $+$
Money supply	$+$ / 0	$+$ / 0	0 / $+$	$+$ / 0	$-$ / 0
Aggregate supply	$-$ / 0	$-$ / 0	0 / $-$	$-$ /$+$	$-$ / $-$

Notes: The symbol "X/Y" indicates that the response of a variable to a shock is X on impact and Y in steady state. Transition from impact to new steady state is monotonic for x, π, and p. An increase in aggregate demand is an increase in the subjective discount rate or a decrease in the marginal utility of wealth. An increase in aggregate supply is an increase in capacity or a decrease in mismatch.

state, the marginal utility for money satisfies $\psi = \delta/(f(x^*) \cdot k) - \omega'(0)$ so a higher k implies a lower ψ, exactly like a negative aggregate demand shock. Hence, π and x necessarily jump down on impact after a positive aggregate supply shock. The response of output is more complicated because $y = f(x) \cdot k$ and x jumps down whereas k jumps up. However, we can exploit the eigenvector $\mathbf{z}$ to prove that the jump of x is always larger than that of k. Hence, y jumps down on impact and increases during the dynamic adjustment toward its new higher steady-state value. The response of i follows because $i = \psi \cdot y$. □

With directed search and costly price adjustment, prices converge slowly toward efficiency. The mechanism is simple. If prices are too high, new markets are created with lower prices but higher tightness. If prices are too low, new markets are created with higher prices but lower tightness. Sellers and buyers have incentives to move to these new markets because they are more efficient so there is a larger surplus to share. Sellers are compensated for the lower price with a higher probability to sell. Buyers are compensated for the higher matching wedge by a lower price. Lazear [2010] finds evidence of such behavior for price and tightness in the US housing market.

This analysis could be used to formalize the conflict between price adjustment and inflation adjustment discussed by Tobin [1993]. In response to a shock, the price adjustment requires an inflation change that further destabilizes the economy, possibly making the recession worse or exacerbating the overheating. This suggests that, even if price adjustments are fairly fast, the temporary inflation changes could amplify short-run fluctuations in tightness and output. For instance, after a negative aggregate demand shock inflation jumps down to allow prices to fall. This fall in prices increase real money balances and stimulates aggregate demand, which eventually brings the economy at efficiency. But a decrease in inflation has a temporary negative effect on aggregate demand and tightness. This negative effect is illustrated in Panel D of Figure 4, where we show that a decrease in inflation depresses the IS curve and thus the AD curve. The economic mechanism is that lower inflation implies higher real interest rates, which lead households to want to accumulate more wealth and hence consume less.

With costly price adjustment, assuming away liquidity traps, monetary policy can accommodate all shocks. A monetary expansion, defined as an increase in money supply, can absorb a negative aggregate demand shock so that output, inflation, and tightness remain at their steady-state level at the time of the shock. Conversely, monetary tightening can absorb a positive aggregate demand shock. A monetary expansion can accommodate a positive aggregate supply shock so that the economy jumps immediately to its new steady state with zero inflation, efficient tightness, and higher output. Conversely, monetary tightening can accommodate a negative supply shock. In all cases, monetary policy should be based on tightness rather than output as efficient output varies with some shocks such as supply shocks while efficient tightness does not.[23]

5.4. The Limit with Zero Price-Adjustment Cost

On the one hand, if the price-adjustment cost is infinite then inflation equals zero. This can be seen in the first-order condition (19), where $\pi(t) = 0$ if $\kappa(t) \to +\infty$. This corresponds to the model studied in Sections 2–4. On the other hand, at the limit without price-adjustment cost, sellers always select a price to maintain the tightness at its efficient level as in Moen [1997]. This can be seen by combining the first-order condition (19), where $\mu(t) = 0$ if $\kappa(t) = 0$, with the first-order condition (20), where $\tau(x(t)) = (1-\eta)/\eta$ if $\mu(t) = 0$; that is, $x(t) = x^*$ when $\kappa(t) = 0$.

This limit without price-adjustment cost describes an economy with flexible prices that always maintain the market for labor services at efficiency. Without price-adjustment cost, market forces drive prices to maintain tightness at efficiency, where consumption is maximized. If tightness were not efficient, both buyers and sellers would be better off with a price adjustment and a cor-

[23] Alternatively, instead of choosing the level of M, the central bank could set the nominal interest rate i to follow an interest-rate rule of the form $i = \alpha \cdot \pi$ with $\alpha > 1$. A negative shock increasing slack leads to negative inflation that prompts the central bank to lower i. Under this monetary policy, the economy immediately adjusts to shocks to remain at $x = x^*$ and $\pi = 0$. New Keynesian models have the same property.

responding tightness adjustment. An implication is that aggregate demand shocks have no impact on tightness or consumption or output when prices are flexible. Aggregate supply shocks have no impact on tightness either, but they have an impact on consumption and output. Effectively, aggregate demand is irrelevant to understand the economy with flexible prices.

With a finite bliss point in the utility for money, $m^* < +\infty$, a large negative aggregate demand shock could bring the economy into a liquidity trap, whereby real money balances are above the bliss point but tightness is still below its efficient level. In that case, the directed search mechanism implies that sellers want to lower their price to increase the tightness they face, even though this does not increase tightness in general equilibrium. The economy may fall into an instantaneous deflationary spiral with no steady-state equilibrium.[24] Hence, liquidity traps are worse with flexible prices than with constant inflation, in line with the results in Werning [2012].

6. Conclusion

In this paper we develop a model of business cycles and use it to study a broad range of stabilization policies. The main attribute of the model is that cyclical fluctuations in demand and supply lead to fluctuations in slack but not in inflation. Our model offers a perspective on business cycles which differs from that of the standard New Keynesian model—in that model fluctuations in demand and supply lead to fluctuations in inflation but not in slack.

Since our model is quite different from the standard New Keynesian model, it offers new insights for the conduct of monetary policy and fiscal policy. A first insight is that market forces do not necessarily push inflation upward when the economy is tight and downward when the economy is slack. Indeed, in a world with matching frictions inflation may remain constant in general equilibrium, so there is no mechanical link between changes in inflation and slack—that is, an accelerationist Phillips curve does not necessarily prevail in equilibrium.[25] The implication is that policymakers should monitor slack directly and not rely on inflation variations as a proxy for slack.

A second insight is that liquidity traps may be permanent and may feature positive inflation

[24] Increasing inflation could push the economy out the trap. However, after the shock has happened and the economy is in a liquidity trap, conventional monetary policy cannot influence inflation anymore even with flexible prices. Helicopter drops or the wealth tax could still successfully pull the economy out of the liquidity trap.

[25] Empirically, it is difficult to describe the sluggish dynamics of inflation with the accelerationist Phillips curve [for example, Ball and Mazumder, 2011; Gordon, 2011; Mankiw and Reis, 2002; Rudd and Whelan, 2007]. Hence, the possibility to explore alternative inflation dynamics in the matching framework is an advantage rather than a drawback.

and high unemployment. In particular, once the economy falls into a liquidity trap, market forces may not be able to bring the economy out of the trap. Since a liquidity trap may be permanent and conventional monetary policy is ineffective in the trap, alternative policies should be implemented to help bring the economy out of the trap. We find that effective policies include a helicopter drop of money, a tax on wealth, or budget-balanced government purchases.

A third insight is that there may not be anything special about a liquidity trap, except for the fact that conventional monetary policy is ineffective. Unlike in a standard New Keynesian model, there are no strange paradoxes in a liquidity trap in our model.[26] Some of the New Keynesian policy recommendations to address liquidity traps would have dire consequences in our model. Hence, more empirical evidence on the effects of policies in a liquidity trap would be valuable before implementing the policies advocated by the New Keynesian model.

A fourth insight is that the departure of tightness from its efficient level is the relevant measure of inefficiency in our model. Since it is relatively easy to measure tightness on the product and labor markets, and since the efficient tightness is unaffected by most shocks, keeping track of the tightness gap is easy and can be nearly done in real time.[27] Even if the efficient tightness varies over time, it is possible to compute measures of the tightness gap from a few sufficient statistics, as explained in Landais, Michaillat and Saez [2010]. Hence, there may be methods to measure economic inefficiency that are more transparent and more responsive than computing output gaps.

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[26] See Eggertsson [2010, 2011] and Eggertsson and Krugman [2012] for a discussion of some of the paradoxes arising in a liquidity trap in a standard New Keynesian model.

[27] Time series for labor market tightness are easily constructed from publicly available monthly data [for example, Shimer, 2005]. Michaillat and Saez [2015*a*] construct time series for product market tightness from publicly available quarterly data.

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