Pricing under Fairness Concerns: Online Appendices

Erik Eyster, Kristóf Madarász, Pascal Michaillat

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Appendix A. Derivations for the Monopoly Model

We derive several of the monopoly results stated in Section 4. In particular, we provide proofs for Lemma 2, Proposition 1, and Corollary 1.

A.1. Properties of the Profit Function

We show that in all the cases treated in Section 4, the first-order condition gives the maximum of the monopoly's profit function. This is because the profit function is unimodal.

The monopoly chooses a price $P > C$ to maximize profits

$$V(P) = (P - C) \cdot Y^d(P).$$

The profit function is differentiable. Its derivative is

$$V'(P) = Y^d + (P - C)\,\frac{dY^d}{dP} = Y^d - (P - C)\,\frac{Y^d}{P}E(P),$$

where

$$E(P) \equiv -\frac{d\ln\left(Y^d\right)}{d\ln(P)} = -\frac{P}{Y^d} \cdot \frac{dY^d}{dP}$$

is the price elasticity of demand. Hence the derivative of the profit function satisfies

$$V'(P) = Y^d(P)\left[1 - \frac{P - C}{P}E(P)\right]. \tag{A.1}$$

We now study the properties of the derivative (A.1) in the various cases considered in Section 4.

No Fairness Concerns. Without fairness concerns, the price elasticity of demand is $E = \epsilon$ (Section 4.3). Hence the derivative (A.1) becomes

$$V'(P) = Y^d(P)\left[1 - \epsilon\frac{P - C}{P}\right].$$

The function $P \mapsto (P - C)/P$ is strictly increasing from 0 to 1 as P increases from C to $+\infty$, so the term in square brackets is strictly decreasing from 1 to $1 - \epsilon < 0$ as P increases from C to $+\infty$. Hence, the term in square brackets has a unique root P^* on $(C, +\infty)$, is positive for $P < P^*$, and is negative for $P > P^*$. Since $Y^d(P) > 0$, these properties transfer to the derivative of the profit function: $V'(P) > 0$ for $P \in (C, P^*)$, $V'(P) = 0$ at $P = P^*$, and $V'(P) < 0$ for $P \in (P^*, +\infty)$. We conclude that the profit function is unimodal, and its maximum P^* is the unique solution to the first-order condition $V'(P) = 0$.

Fairness Concerns and Observable Costs. With fairness concerns and observable costs, the price elasticity of demand is $E = \epsilon + (\epsilon - 1)\phi(P/C)$ (Section 4.4). The profit function is now defined for $P \in (C, M^h \cdot C)$. The derivative (A.1) becomes

$$V'(P) = Y^d(P)\left[1 - \frac{P - C}{P} \cdot \{\epsilon + (\epsilon - 1)\phi(P/C)\}\right].$$

Again, the function $P \mapsto (P - C)/P$ is strictly increasing from 0 to 1 as P increases from C to $+\infty$. The elasticity of the fairness function $\phi(P/C)$ is strictly increasing from $\phi(1) > 0$ to $+\infty$ as P increases from C to $M^h \cdot C$ (Lemma 2). Hence the term in square brackets is strictly decreasing from 1 to $-\infty$ as P increases from C to $M^h \cdot C$. This implies that the term in square brackets has a unique root P^* on $(C, M^h \cdot C)$, is positive for $P < P^*$, and is negative for $P > P^*$. Following the same argument as in the previous case, we conclude that the profit function is unimodal, and its maximum P^* is the unique solution to the first-order condition $V'(P) = 0$.

Fairness Concerns and Rational Inference of Costs. With fairness concerns and rational inference of marginal costs, the price elasticity of demand is again $E = \epsilon$ (Section 4.5). Hence, as in the case of no fairness concerns, the profit function is unimodal and its maximum is the unique solution to the first-order condition $V'(P) = 0$.

Fairness Concerns and Subproportional Inference of Costs. With fairness concerns and subproportional inference of costs, the price elasticity of demand is $E = \epsilon + (\epsilon - 1)\gamma\phi(M^p(P))$ (Section 4.6). The profit function is now defined for $P \in (C, P^b)$, where the upper bound is defined by

$$P^b = \frac{\epsilon}{\epsilon - 1}(M^h)^{1/\gamma} C^b. \tag{A.2}$$

The price P^b is such that at P^b, the perceived markup reaches the upper bound of the domain of the fairness function: $M^p(P^b) = M^h$. We know that $P^b > C$ because $C^b > (\epsilon - 1) \cdot (M^h)^{-1/\gamma} \cdot C/\epsilon$ (Definition 3). The derivative (A.1) becomes

$$V'(P) = Y^d(P)\left[1 - \frac{P - C}{P} \cdot \left\{\epsilon + (\epsilon - 1)\gamma\phi(M^p(P))\right\}\right].$$

Again, the function $P \mapsto (P - C)/P$ is strictly increasing from 0 to 1 as P increases from C to $+\infty$. The perceived markup $M^p(P)$ is strictly increasing from $M^p(C) > 0$ to M^h as P increases from C to P^b (Lemma 5). Hence, the elasticity of the fairness function $\phi(M^p(P))$ is strictly increasing from $\phi(M^p(C)) > 0$ to $+\infty$ as P increases from C to P^b (Lemma 2). Since $\gamma > 0$, we infer that the term in square brackets is strictly decreasing from 1 to $-\infty$ as P increases from C to P^b. Thus the term in square brackets has a unique root P^* on (C, P^b), is positive for $P < P^*$, and is negative for $P > P^*$. Following the same argument as in the previous cases, we conclude that the profit function is unimodal, and its maximum P^* is the unique solution to the first-order condition $V'(P) = 0$.

A.2. Proof of Lemma 2

By definition, the elasticity of the fairness function is given by

$$\phi(M^p) = -M^p \cdot \frac{F'(M^p)}{F(M^p)}.$$

The properties of the fairness function F listed in Definition 2 indicate that $F(M^p) > 0$ and $F'(M^p) < 0$, so $\phi(M^p) > 0$.

The properties also indicate that $F > 0$ is decreasing in M^p, and that $F' < 0$ is decreasing in M^p (as F is concave in M^p). Thus, both $1/F > 0$ and $-F' > 0$ are increasing in M^p, which implies that ϕ is strictly increasing in M^p.

Next, the properties indicate that $F(0) > 0$ and $F'(0)$ is finite, so $\lim_{M^p \to 0} \phi(M^p) = 0$. And they indicate that $F(M^h) = 0$ while $M^h > 0$ and $F'(M^h) < 0$, so $\lim_{M^p \to M^h} \phi(M^p) = +\infty$.

Last, the superelasticity of the fairness function is given by

$$\sigma = M^p \cdot \frac{\phi'(M^p)}{\phi(M^p)}.$$

Since $\phi(M^p) > 0$ and $\phi'(M^p) > 0$, it is clear that $\sigma > 0$.

A.3. Proof of Proposition 1

Markup. Since customers care about fairness and infer subproportionally, the price elasticity of demand is $E = \epsilon + (\epsilon - 1)\gamma\phi(M^p(P))$. Moreover, the monopoly's optimal markup is

$$M = \frac{E}{E-1} = 1 + \frac{1}{E-1}.$$

Combining these equations yields the markup

$$M = 1 + \frac{1}{\epsilon - 1} \cdot \frac{1}{1 + \gamma\phi(M^p(M \cdot C))}. \tag{A.3}$$

In (A.3) we have used the relationship between the price and markup: $P = M \cdot C$.

Toward showing that (A.3) admits a unique solution, we introduce the price P^b defined by (A.2) and the markup $M^b = P^b/C > 1$. Since $P = M \cdot C$, P strictly increases from 0 to P^b when M increases from 0 to M^b. Next, Lemma 5 shows that $M^p(P)$ strictly increases from 0 to M^h when P increases from 0 to P^b. Last, Lemma 2 indicates that $\phi(M^p)$ strictly increases from 0 to ∞ when M^p increases from 0 to M^h. As $\gamma > 0$, we conclude that when M increases from 0 to $M^b > 1$, the right-hand side of (A.3) strictly decreases from $\epsilon/(\epsilon-1)$ to 1. Hence, (A.3) has a unique solution $M \in \left[0, M^b\right]$, implying that the markup exists and unique. Given the range of values taken by the right-hand side of (A.3), we also infer that $M \in (1, \epsilon/(\epsilon-1))$.

Passthrough. We now compute the cost passthrough, $\beta = d\ln(P)/d\ln(C)$. The equilibrium price is $P = M(M^p(P)) \cdot C$, where the markup $M(M^p)$ is given by (A.3). Using this price equation, we obtain

$$\beta = \frac{d\ln(M)}{d\ln(M^p)} \cdot \frac{d\ln(M^p)}{d\ln(P)} \cdot \frac{d\ln(P)}{d\ln(C)} + 1.$$

Since $d\ln(M^p)/d\ln(P) = \gamma$ (Lemma 5) and $d\ln(P)/d\ln(C) = \beta$ (by definition), we get

$$\beta = \frac{1}{1 - \gamma \frac{d\ln(M)}{d\ln(M^p)}}. \tag{A.4}$$

Our next step is to compute the elasticity of $M(M^p)$ with respect to M^p from (A.3):

$$-\frac{d\ln(M)}{d\ln(M^p)} = -\frac{1}{M} \cdot \frac{dM}{d\ln(M^p)} = \frac{1}{M} \cdot \frac{1}{\epsilon - 1} \cdot \frac{1}{1+\gamma\phi} \cdot \frac{1}{1+\gamma\phi} \cdot \gamma \cdot \frac{d\phi}{d\ln(M^p)}.$$

Using (A.3), we find that

$$(\epsilon - 1)(1+\gamma\phi)M = \epsilon + (\epsilon - 1)\gamma\phi.$$

Moreover, by definition, the superelasticity σ of the fairness function satisfies $\phi\sigma = d\phi/d\ln(M^p)$. Combining these three results, we obtain

$$-\frac{d\ln(M)}{d\ln(M^p)} = \frac{\gamma\phi\sigma}{[\epsilon + (\epsilon-1)\gamma\phi](1+\gamma\phi)}. \tag{A.5}$$

Finally, combining (A.4) with (A.5) yields the cost passthrough

$$\beta = \frac{1}{1 + \frac{\gamma^2\phi\sigma}{(1+\gamma\phi)[\epsilon+(\epsilon-1)\gamma\phi]}}. \tag{A.6}$$

Since $\gamma > 0$ (Definition 3), $\phi > 0$ (Lemma 2), and $\sigma > 0$ (also Lemma 2), we infer that $\beta \in (0, 1)$.

A.4. Proof of Corollary 1

We apply the results of Proposition 1 to a specific fairness function:

$$F(M^p) = 1 - \theta \cdot \left(M^p - \frac{\epsilon}{\epsilon - 1}\right). \tag{A.7}$$

We also assume that customers are acclimated, so $M^p = \epsilon/(\epsilon - 1)$ and $F = 1$.

Preliminary Results. The elasticity of the fairness function (A.7) is

$$\phi = -\frac{M^p}{F} \cdot \frac{dF}{dM^p} = \frac{M^p}{F} \cdot \theta.$$

Accordingly, the superelasticity of the fairness function (A.7) satisfies

$$\sigma = \frac{d\ln(\phi)}{d\ln(M^p)} = 1 - \frac{d\ln(F)}{d\ln(M^p)} = 1 + \phi.$$

When $M^p = \epsilon/(\epsilon - 1)$ and $F = 1$, the elasticity and superelasticity simplify to

$$\phi = \frac{\epsilon\theta}{\epsilon - 1} \tag{A.8}$$

$$\sigma = 1 + \frac{\epsilon\theta}{\epsilon - 1}. \tag{A.9}$$

Markup. Combining (A.3) with (A.8), we obtain the following markup:

$$M = 1 + \frac{1}{\epsilon - 1} \cdot \frac{1}{1 + \gamma\epsilon\theta/(\epsilon - 1)} = 1 + \frac{1}{(1 + \gamma\theta)\epsilon - 1}.$$

This expression shows that M is lower when ϵ, γ, or θ are higher.

Passthrough. Combining (A.6) with (A.8) and (A.9), we find that the cost passthrough β satisfies

$$1/\beta = 1 + \frac{\gamma^2\epsilon\theta\left[(\epsilon - 1) + \epsilon\theta\right]}{(\epsilon - 1)\left[(\epsilon - 1) + \gamma\epsilon\theta\right](\epsilon + \gamma\epsilon\theta)} = 1 + \frac{\gamma^2\theta\left[(1 + \theta)\epsilon - 1\right]}{(\epsilon - 1)\left[(1 + \gamma\theta)\epsilon - 1\right](1 + \gamma\theta)}.$$

Next we introduce the auxiliary function

$$\Delta(\gamma, \theta, \epsilon) = \frac{\gamma^2\theta\left[(1 + \theta)\epsilon - 1\right]}{(\epsilon - 1)\left[(1 + \gamma\theta)\epsilon - 1\right](1 + \gamma\theta)}, \tag{A.10}$$

where $\gamma \in (0, 1]$, $\theta > 0$, and $\epsilon > 1$.

First, we divide numerator and denominator of Δ by $(\epsilon - 1)$:

$$\Delta(\gamma, \theta, \epsilon) = \frac{\gamma^2\theta\,[1 + \theta\epsilon/(\epsilon - 1)]}{[(1 + \gamma\theta)\epsilon - 1]\,(1 + \gamma\theta)}.$$

Since $\epsilon/(\epsilon - 1)$ is decreasing in ϵ and $(1 + \gamma\theta)\epsilon - 1$ is increasing in ϵ, Δ is decreasing in ϵ. As $\beta = 1/(1 + \Delta)$, we conclude that β is increasing in ϵ.

Next, we divide numerator and denominator of Δ in (A.10) by $\theta\,(\epsilon\theta + \epsilon - 1)$:

$$\Delta(\gamma, \theta, \epsilon) = \frac{\gamma^2}{(\epsilon - 1)\,(\gamma + 1/\theta)\,\frac{\gamma\epsilon\theta+\epsilon-1}{\epsilon\theta+\epsilon-1}}.$$

First, $\gamma + 1/\theta$ is decreasing in θ. Second, as $\epsilon > 1$ and $\gamma \leq 1$, $(\gamma\epsilon\theta + \epsilon - 1)/(\epsilon\theta + \epsilon - 1)$ is decreasing in θ. Hence, Δ is increasing in θ, and as $\beta = 1/(1 + \Delta)$, β is decreasing in θ.

Last, dividing numerator and denominator of Δ in (A.10) by γ^2, we get

$$\Delta(\gamma, \theta, \epsilon) = \frac{\theta\,[(1 + \theta)\epsilon - 1]}{(\epsilon - 1)\,[\theta\epsilon + (\epsilon - 1)/\gamma]\,(\theta + 1/\gamma)}.$$

The denominator is decreasing in γ, so Δ is increasing in γ. Since $\beta = 1/(1 + \Delta)$, we conclude that β is decreasing in γ.

Appendix B. Derivations for the New Keynesian Model

We derive the properties of the New Keynesian model with fairness presented in Section 5. In particular, we prove Lemmas 6 and 7, Propositions 2 and 3, and Corollary 2.

B.1. Household and Firm Problems

We begin by solving the problems of households and firms.

Household k's Problem. To solve household k's problem, we set up the Lagrangian:

$$\begin{aligned}\mathcal{L}_k = \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \Bigg\{ &\ln(Z_k(t)) - \frac{N_k(t)^{1+\eta}}{1+\eta} \\ &+ \mathcal{A}_k(t) \left[W_k(t)N_k(t) + B_k(t-1) + V_k(t) - Q(t)B_k(t) - \int_0^1 P_j(t)Y_{jk}(t)\, dj \right] \\ &+ \mathcal{B}_k(t) \left[N_k^d(t, W_k(t)) - N_k(t) \right] \Bigg\}.\end{aligned}$$

In the Lagrangian, $\mathcal{A}_k(t)$ is the Lagrange multiplier on the budget constraint in period t; $\mathcal{B}_k(t)$ is the Lagrange multiplier on the labor-demand constraint in period t; and $Z_k(t)$ is the fairness-adjusted consumption index:

$$Z_k(t) = \left[\int_0^1 Z_{jk}(t)^{(\epsilon-1)/\epsilon}\, dj \right]^{\epsilon/(\epsilon-1)}. \tag{B.1}$$

In the consumption index, $Z_{jk}(t)$ is the fairness-adjusted consumption of good j:

$$Z_{jk}(t) = F_j(t) \cdot Y_{jk}(t). \tag{B.2}$$

First-Order Conditions with Respect to Consumption. We first compute the first-order conditions with respect to $Y_{jk}(t)$ for all goods $j \in [0,1]$: $\partial \mathcal{L}_k / \partial Y_{jk}(t) = 0$. From the definitions of $Z_k(t)$ and $Z_{jk}(t)$ given by (B.1) and (B.2), we find

$$\frac{\partial Z_{jk}(t)}{\partial Y_{jk}(t)} = F_j(t) \quad \text{and} \quad \frac{\partial Z_k(t)}{\partial Z_{jk}(t)} = \left[\frac{Z_{jk}(t)}{Z_k(t)} \right]^{-1/\epsilon} dj.$$

Hence the first-order conditions imply that for all $j \in [0,1]$,

$$\left[\frac{Z_{jk}(t)}{Z_k(t)}\right]^{-1/\epsilon} \frac{F_j(t)}{Z_k(t)} = \mathcal{A}_k(t)P_j(t). \tag{B.3}$$

Taking (B.3) to the power of $1-\epsilon$ and shuffling terms, we then obtain

$$\frac{1}{Z_k(t)^{1-\epsilon}} \cdot \frac{1}{Z_k(t)^{(\epsilon-1)/\epsilon}} \cdot Z_{jk}(t)^{(\epsilon-1)/\epsilon} = \mathcal{A}_k(t)^{1-\epsilon}\left[\frac{P_j(t)}{F_j(t)}\right]^{1-\epsilon}.$$

We integrate this equation over $j \in [0,1]$, use the definition of $Z_k(t)$ given by (B.1), and introduce the price index

$$X(t) = \left\{\int_0^1 \left[\frac{P_j(t)}{F_j(t)}\right]^{1-\epsilon} dj\right\}^{1/(1-\epsilon)}. \tag{B.4}$$

We obtain the following:

$$\frac{1}{Z_k(t)^{1-\epsilon}} \cdot \frac{Z_k(t)^{(\epsilon-1)/\epsilon}}{Z_k(t)^{(\epsilon-1)/\epsilon}} = \mathcal{A}_k(t)^{1-\epsilon}X(t)^{1-\epsilon}.$$

From this equation we infer

$$\mathcal{A}_k(t) = \frac{1}{X(t)Z_k(t)}. \tag{B.5}$$

Last, combining (B.3) and (B.5), we find that the optimal fairness-adjusted consumption of good j by household k satisfies

$$Z_{jk}(t) = Z_k(t)\left[\frac{P_j(t)}{X(t)}\right]^{-\epsilon} F_j(t)^{\epsilon}.$$

As consumption and fairness-adjusted consumption of good j are related by $Y_{jk}(t) = Z_{jk}(t)/F_j(t)$, the optimal consumption of good j by household k satisfies

$$Y_{jk}(t) = Z_k(t)\left[\frac{P_j(t)}{X(t)}\right]^{-\epsilon} F_j(t)^{\epsilon-1}. \tag{B.6}$$

Integrating (B.6) over all households $k \in [0,1]$ yields the output of good j:

$$Y_j(t) = Z(t)\left[\frac{P_j(t)}{X(t)}\right]^{-\epsilon} F_j(t)^{\epsilon-1}.$$

We note that the fairness factor $F_j(t)$ is a function of the perceived price markup, $F_j(t) = F_j(P_j(t)/C_j^p(t))$, and that the perceived marginal cost $C_j^p(t)$ follows the law of motion (8). These observations allow us to obtain the demand for good j:

$$Y_j^d(t, P_j(t), C_j^p(t-1)) = Z(t)\left[\frac{P_j(t)}{X(t)}\right]^{-\epsilon} F_j\left(\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\gamma}\left[\frac{P_j(t)}{C_j^p(t-1)}\right]^{\gamma}\right)^{\epsilon-1}.$$

For future reference, the elasticities of the function $Y_j^d(t, P_j(t), C_j^p(t-1))$ are

$$-\frac{\partial \ln\left(Y_j^d\right)}{\partial \ln\left(P_j\right)} = \epsilon + (\epsilon-1)\gamma\phi_j(M_j^p(t)) \equiv E_j(M_j^p(t)) \tag{B.7}$$

$$\frac{\partial \ln\left(Y_j^d\right)}{\partial \ln\left(C_j^p\right)} = (\epsilon-1)\gamma\phi_j(M_j^p(t)) = E_j(M_j^p(t)) - \epsilon, \tag{B.8}$$

where $\phi_j = -d\ln(F_j)/d\ln(M_j^p)$ is the elasticity of the fairness function. The function E_j gives the price elasticity of the demand for good j.

Moreover, using (B.6) and the definition of the price index X given by (B.4), we find that

$$\int_0^1 P_j Y_{jk}\, dj = X^{\epsilon} Z_k \int_0^1 \left(\frac{P_j}{F_j}\right)^{1-\epsilon} dj = XZ_k.$$

This means that when households optimally allocate their consumption expenditures across goods, the price of one unit of fairness-adjusted consumption index is X.

First-Order Condition with Respect to Bonds. The first-order condition with respect to $B_k(t)$ is $\partial\mathcal{L}_k/\partial B_k(t) = 0$, which gives

$$Q(t)\mathcal{A}_k(t) = \delta\,\mathbb{E}_t(\mathcal{A}_k(t+1)).$$

Using (B.5), we obtain household k's consumption Euler equation:

$$Q(t) = \delta\,\mathbb{E}_t\left(\frac{X(t)Z_k(t)}{X(t+1)Z_k(t+1)}\right). \tag{B.9}$$

This equation governs how the household smooths fairness-adjusted consumption over time.

Firm j's Problem. Since the wages set by households depend on firms' labor demands, we turn to the firms' problems before finishing the households' problems. To solve firm j's problem, we set up the Lagrangian:

$$\begin{aligned}\mathcal{L}_j = \mathbb{E}_0 \sum_{t=0}^{\infty} \Gamma(t) \Bigg\{ & P_j(t) Y_j(t) - \int_0^1 W_k(t) N_{jk}(t)\, dk \\ & + \mathcal{H}_j(t) \left[Y_j^d(t, P_j(t), C_j^p(t-1)) - Y_j(t) \right] + \mathcal{J}_j(t) \left[A_j(t) N_j(t)^{\alpha} - Y_j(t) \right] \\ & + \mathcal{K}_j(t) \left[C_j^p(t-1)^{\gamma} \left[\frac{\epsilon - 1}{\epsilon} P_j(t) \right]^{1-\gamma} - C_j^p(t) \right] \Bigg\}.\end{aligned}$$

In the Lagrangian, $\mathcal{H}_j(t)$ is the Lagrange multiplier on the demand constraint in period t; $\mathcal{J}_j(t)$ is the Lagrange multiplier on the production constraint in period t; $\mathcal{K}_j(t)$ is the Lagrange multiplier on the law of motion of the perceived marginal cost in period t; and $N_j(t)$ is the employment index:

$$N_j(t) = \left[\int_0^1 N_{jk}(t)^{(\nu-1)/\nu}\, dk \right]^{\nu/(\nu-1)}. \tag{B.10}$$

First-Order Conditions with Respect to Employment. We compute the first-order conditions with respect to $N_{jk}(t)$ for all labor services $k \in [0, 1]$: $\partial \mathcal{L}_j / \partial N_{jk}(t) = 0$. From the definition of $N_j(t)$ given by (B.10), we know that

$$\frac{\partial N_j(t)}{\partial N_{jk}(t)} = \left[\frac{N_{jk}(t)}{N_j(t)} \right]^{-1/\nu} dk.$$

Hence the first-order conditions imply that for all $k \in [0, 1]$,

$$W_k(t) = \alpha \mathcal{J}_j(t) A_j(t) N_j(t)^{\alpha-1} \left[\frac{N_{jk}(t)}{N_j(t)} \right]^{-1/\nu}. \tag{B.11}$$

Toward deriving firm j's labor demand, we introduce the wage index

$$W(t) = \left[\int_0^1 W_k(t)^{1-\nu}\, dk \right]^{1/(1-\nu)}. \tag{B.12}$$

Taking (B.11) to the power of $1 - \nu$, we obtain

$$W_k(t)^{1-\nu} = \left[\alpha \mathcal{J}_j(t) A_j(t) N_j(t)^{\alpha-1} \right]^{1-\nu} \frac{1}{N_j(t)^{(\nu-1)/\nu}} N_{jk}(t)^{(\nu-1)/\nu}.$$

Integrating this condition over $k \in [0, 1]$ and using the definitions of N_j and W given by (B.10) and (B.12), we find

$$W(t)^{1-\nu} = \left[\alpha \mathcal{J}_j(t) A_j(t) N_j(t)^{\alpha-1}\right]^{1-\nu} \frac{N_j(t)^{(\nu-1)/\nu}}{N_j(t)^{(\nu-1)/\nu}}.$$

From this equation we infer

$$W(t) = \alpha \mathcal{J}_j(t) A_j(t) N_j(t)^{\alpha-1}. \tag{B.13}$$

Last, we combine (B.11) and (B.13) to determine the quantity of labor that firm j hires from household k:

$$N_{jk}(t) = N_j(t) \left[\frac{W_k(t)}{W(t)}\right]^{-\nu}. \tag{B.14}$$

Integrating (B.14) over all firms $j \in [0, 1]$ yields the demand for labor service k:

$$N_k^d(t, W_k(t)) = N(t) \left[\frac{W_k(t)}{W(t)}\right]^{-\nu}, \tag{B.15}$$

where $N(t) = \int_0^1 N_j(t)\, dj$ is aggregate employment.

Moreover, (B.12) and (B.14) imply that

$$\int_0^1 W_k N_{jk}\, dk = W^\nu N_j \int_0^1 W_k^{1-\nu}\, dk = W N_j.$$

This means that when firms optimally allocate their wage bill across labor services, the cost of one unit of labor index is W.

First-Order Conditions with Respect to Labor and Wage. We now finish solving household k's problem using labor demand (B.15). The first-order conditions with respect to $N_k(t)$ and $W_k(t)$ are $\partial \mathcal{L}_k / \partial N_k(t) = 0$ and $\partial \mathcal{L}_k / \partial W_k(t) = 0$; they yield

$$N_k(t)^\eta = \mathcal{A}_k(t) W_k(t) - \mathcal{B}_k(t) \tag{B.16}$$

$$\mathcal{A}_k(t) N_k(t) = -\mathcal{B}_k(t) \frac{dN_k^d}{dW_k}. \tag{B.17}$$

Since the elasticity of N_k^d with respect to W_k is $-\nu$, we infer from (B.17) that

$$\mathcal{A}_k(t) W_k(t) = \mathcal{B}_k(t) \nu. \tag{B.18}$$

Plugging this result into (B.16), we obtain

$$\mathcal{B}_k(t) = \frac{N_k(t)^\eta}{\nu - 1}.$$

Combining this result with (B.18) then yields

$$W_k(t) = \frac{\nu}{\nu - 1} \cdot \frac{N_k(t)^\eta}{\mathcal{A}_k(t)}.$$

Finally, by merging this equation with (B.5), we find that household k sets its wage rate at

$$\frac{W_k(t)}{X(t)} = \frac{\nu}{\nu - 1} N_k(t)^\eta Z_k(t). \tag{B.19}$$

This equation shows that households set their real wage at a markup of $\nu/(\nu - 1) > 1$ over the marginal rate of substitution between leisure and consumption.

First-Order Condition with Respect to Output. We then finish solving firm j's problem. The first-order condition with respect to $Y_j(t)$ is $\partial \mathcal{L}_j / \partial Y_j(t) = 0$, which gives

$$P_j(t) = \mathcal{H}_j(t) + \mathcal{J}_j(t).$$

Using the value of $\mathcal{J}_j(t)$ given by (B.13), we then obtain

$$\mathcal{H}_j(t) = P_j(t) \left[1 - \frac{W(t)/P_j(t)}{\alpha A_j(t) N_j(t)^{\alpha - 1}} \right]. \tag{B.20}$$

Firm j's nominal marginal cost is the nominal wage divided by the marginal product of labor:

$$C_j(t) = \frac{W(t)}{\alpha A_j(t) N_j(t)^{\alpha - 1}}. \tag{B.21}$$

Hence the first-order condition (B.20) can be written

$$\mathcal{H}_j(t) = P_j(t) \left[1 - \frac{C_j(t)}{P_j(t)} \right]. \tag{B.22}$$

Given that firm j's markup is $M_j(t) = P_j(t)/C_j(t)$, we rewrite this equation as

$$\frac{\mathcal{H}_j(t)}{P_j(t)} = \frac{M_j(t) - 1}{M_j(t)}. \tag{B.23}$$

First-Order Condition with Respect to Price. The first-order condition of firm j's problem with respect to $P_j(t)$ is $\partial \mathcal{L}_j / \partial P_j(t) = 0$. It yields

$$0 = Y_j(t) + \mathcal{H}_j(t) \frac{\partial Y_j^d}{\partial P_j} + (1-\gamma) \mathcal{K}_j(t) \frac{C_j^p(t)}{P_j(t)}. \tag{B.24}$$

We divide this condition by $Y_j(t)$ and insert the price elasticity of the demand for good j, $E_j(M_j^p(t)) = -\partial \ln(Y_j^d)/\partial \ln(P_j)$, as well as the perceived price markup for good j, $M_j^p(t) = P_j(t)/C_j^p(t)$. We obtain

$$0 = 1 - \frac{\mathcal{H}_j(t) E_j(M_j^p(t))}{P_j(t)} + (1-\gamma) \frac{\mathcal{K}_j(t)}{Y_j(t) M_j^p(t)}.$$

Using the value of $\mathcal{H}_j(t)$ given by (B.23), we finally obtain

$$(1-\gamma) \frac{\mathcal{K}_j(t)}{Y_j(t) M_j^p(t)} = \frac{M_j(t) - 1}{M_j(t)} E_j(M_j^p(t)) - 1. \tag{B.25}$$

First-Order Condition with Respect to Perceived Marginal Cost. The first-order condition of firm j's problem with respect to $C_j^p(t)$ is $\partial \mathcal{L}_j / \partial C_j^p(t) = 0$. It gives

$$0 = \mathbb{E}_t \left(\frac{\Gamma(t+1)}{\Gamma(t)} \mathcal{H}_j(t+1) \frac{\partial Y_j^d}{\partial C_j^p} \right) + \gamma \, \mathbb{E}_t \left(\frac{\Gamma(t+1)}{\Gamma(t)} \mathcal{K}_j(t+1) \frac{C_j^p(t+1)}{C_j^p(t)} \right) - \mathcal{K}_j(t).$$

And using the elasticity given by (B.8), we find

$$\mathcal{K}_j(t) = \mathbb{E}_t \left(\frac{\Gamma(t+1)}{\Gamma(t)} \left\{ \mathcal{H}_j(t+1) \frac{Y_j(t+1)}{C_j^p(t)} [E_j(M_j^p(t+1)) - \epsilon] + \gamma \mathcal{K}_j(t+1) \frac{C_j^p(t+1)}{C_j^p(t)} \right\} \right).$$

We modify this equation in two steps: first, we multiply it by $C_j^p(t)/[Y_j(t) P_j(t)]$; second, we insert the perceived price markups $M_j^p(t) = P_j(t)/C_j^p(t)$ and $M_j^p(t+1) = P_j(t+1)/C_j^p(t+1)$. We get

$$\frac{\mathcal{K}_j(t) M_j^p(t)}{Y_j(t)} = \mathbb{E}_t \left(\frac{\Gamma(t+1) Y_j(t+1) P_j(t+1)}{\Gamma(t) Y_j(t) P_j(t)} \left\{ \frac{\mathcal{H}_j(t+1)}{P_j(t+1)} [E_j(M_j^p(t+1)) - \epsilon] + \gamma \frac{\mathcal{K}_j(t+1) M_j^p(t+1)}{Y_j(t+1)} \right\} \right).$$

Last, we multiply the equation by $(1-\gamma)$; and we eliminate $\mathcal{H}_j(t+1)$ using (B.23) and $\mathcal{K}_j(t)$ and $\mathcal{K}_j(t+1)$ using (B.25). We obtain firm j's pricing equation, which links its markup to its perceived markup:

$$\text{(B.26)} \qquad \frac{M_j(t)-1}{M_j(t)} E_j(M_j^p(t)) = 1 + \mathbb{E}_t\left(\frac{\Gamma(t+1)Y_j(t+1)P_j(t+1)}{\Gamma(t)Y_j(t)P_j(t)}\left\{\frac{M_j(t+1)-1}{M_j(t+1)}[E_j(M_j^p(t+1)) - (1-\gamma)\epsilon] - \gamma\right\}\right).$$

B.2. Equilibrium

We present the equilibrium of the model. Because all households and firms face the same conditions, they all behave the same in equilibrium, so we drop the subscripts j and k on all variables.

The equilibrium can be described by seven variables: output $Y(t)$, employment $N(t)$, the price level $P(t)$, the wage $W(t)$, the bond price $Q(t)$, the price markup $M(t)$, and the perceived price markup $M^p(t)$. Seven equations determine these seven variables.

The first equation is the monetary-policy rule, given by (10). This equation links the nominal interest rate, $i(t)$, to the inflation rate, $\pi(t)$. By definition, however, the nominal interest rate is determined by the bond price and the inflation rate by the price level:

$$\text{(B.27)} \qquad i(t) = \ln\left(\frac{1}{Q(t)}\right), \qquad \pi(t) = \ln\left(\frac{P(t)}{P(t-1)}\right).$$

Hence the monetary-policy rule links bond price to price level.

The second equation is the production function, which is directly obtained from (9):

$$\text{(B.28)} \qquad Y(t) = A(t)N(t)^{\alpha}.$$

The third equation is the usual consumption Euler equation, obtained by simplifying (B.9). By symmetry $X(t) = P(t)/F(t)$ and $Z_k(t) = F(t)Y(t)$, so (B.9) gives

$$\text{(B.29)} \qquad Q(t) = \delta\,\mathbb{E}_t\left(\frac{P(t)Y(t)}{P(t+1)Y(t+1)}\right).$$

The fourth equation is the usual expression for the real wage, obtained by simplifying (B.19). Once again, by symmetry $X(t) = P(t)/F(t)$ and $Z_k(t) = F(t)Y(t)$, so (B.19) yields

$$\frac{W(t)}{P(t)} = \frac{\nu}{\nu-1} N(t)^{\eta} Y(t).$$

Combining this equation with (B.28), we express the real wage as a function of employment:

$$\frac{W(t)}{P(t)} = \frac{\nu}{\nu - 1} A(t) N(t)^{\eta+\alpha}. \tag{B.30}$$

The fifth equation is the standard link between price markup and employment, which is obtained from the definition of the price markup. In a symmetric economy the price markup is just the inverse of the real marginal cost: $M(t) = P(t)/C(t)$. Combining the expression of the nominal marginal cost given by (B.21) with the value of the real wage given by (B.30), we infer the real marginal cost:

$$\frac{C(t)}{P(t)} = \frac{\nu}{(\nu - 1)\alpha} N(t)^{1+\eta}.$$

Since the price markup is the inverse of the real marginal cost, we find

$$N(t) = \left[\frac{(\nu - 1)\alpha}{\nu} \cdot \frac{1}{M(t)}\right]^{1/(1+\eta)}. \tag{B.31}$$

The sixth equation is a pricing equation, which is obtained by simplifying (B.26). In equilibrium the stochastic discount factor is given by

$$\Gamma(t) = \delta^t \cdot \frac{X(0)Z(0)}{X(t)Z(t)}.$$

Since by symmetry $Z(t) = F(t)Y(t)$ and $X(t) = P(t)/F(t)$, we have

$$\frac{\Gamma(t+1)}{\Gamma(t)} = \delta \cdot \frac{X(t)}{X(t+1)} \cdot \frac{Z(t)}{Z(t+1)} = \delta \cdot \frac{P(t)}{P(t+1)} \cdot \frac{Y(t)}{Y(t+1)}.$$

Hence, (B.26) simplifies to

$$\frac{M(t) - 1}{M(t)} E(M^p(t)) = 1 - \delta\gamma + \delta\, \mathbb{E}_t\left(\frac{M(t+1) - 1}{M(t+1)} \left[E(M^p(t+1)) - (1-\gamma)\epsilon\right]\right). \tag{B.32}$$

This pricing equation shows the dynamic relationship between actual and perceived price markups. Unlike the other equilibrium conditions—which are the same as in the textbook model—the pricing equation is unique to the model with fairness.

The seventh and final equation is the law of motion of the perceived price markup. It derives from the law of motion of the perceived marginal cost, given by (8). Since $M^p(t) = P(t)/C^p(t)$, (8) implies

$$M^p(t) = \left[\frac{P(t)}{(\epsilon-1)P(t)/\epsilon}\right]^{1-\gamma}\left[\frac{P(t)}{C^p(t-1)}\right]^{\gamma} = \left(\frac{\epsilon}{\epsilon-1}\right)^{1-\gamma}\left[\frac{P(t)}{P(t-1)}\right]^{\gamma}\left[\frac{P(t-1)}{C^p(t-1)}\right]^{\gamma}.$$

Hence the perceived price markup satisfies

$$M^p(t) = \left(\frac{\epsilon}{\epsilon-1}\right)^{1-\gamma}\left[\frac{P(t)}{P(t-1)}\right]^{\gamma}\left[M^p(t-1)\right]^{\gamma}. \tag{B.33}$$

B.3. Steady-State Equilibrium

We now apply the equilibrium conditions to a steady-state environment, in which all real variables are constant and all nominal variables grow at the inflation rate, $\overline{\pi}$. We use these steady-state conditions to prove Lemma 7, Proposition 3, and Corollary 2. We also use these conditions to compute the long-run Phillips curves that are displayed in Figure 3.

We describe the steady-state equilibrium by six variables: output $\overline{Y}$, employment $\overline{N}$, inflation $\overline{\pi}$, nominal interest rate $\overline{i}$, price markup $\overline{M}$, and perceived price markup $\overline{M^p}$. These six variables are governed by six equations.

Steady-State Equilibrium Conditions. First, in steady state the consumption Euler equation (B.29) gives

$$\overline{Q} = \delta \cdot \frac{P(t)}{P(t+1)}.$$

Taking the logarithm of this equation and using (B.27), we obtain

$$\overline{i} = \rho + \overline{\pi},$$

where $\rho \equiv -\ln(\delta)$ is the discount rate. Equivalently, the steady-state real interest rate $\overline{r} \equiv \overline{i} - \overline{\pi}$ equals the discount rate ρ.

Second, in steady state the monetary-policy rule (10) implies that $\overline{r} = \overline{i_0} + (\psi - 1)\overline{\pi}$. Since $\overline{r} = \rho$, the steady-state inflation rate is

$$\overline{\pi} = \frac{\rho - \overline{i_0}}{\psi - 1}.$$

Third, in steady state the law of motion of the perceived price markup (B.33) implies that

$$(\overline{M^p})^{1-\gamma} = \left(\frac{\epsilon}{\epsilon-1}\right)^{1-\gamma}\left[\frac{P(t)}{P(t-1)}\right]^{\gamma}.$$

Taking this expression to the power of $1/(1-\gamma)$, and noting that in steady state $P(t)/P(t-1) = \exp(\overline{\pi})$, we find that the steady-state perceived price markup is

$$\text{(B.34)} \qquad \overline{M^p} = \frac{\epsilon}{\epsilon - 1} \exp\left(\frac{\gamma}{1-\gamma}\overline{\pi}\right).$$

Fourth, in steady state the pricing equation (B.32) implies that

$$\text{(B.35)} \qquad 0 = 1 - \delta\gamma - \frac{\overline{M}-1}{\overline{M}} E(\overline{M^p}) + \delta \frac{\overline{M}-1}{\overline{M}} \left[E(\overline{M^p}) - (1-\gamma)\epsilon\right].$$

Shuffling this expression, we obtain the following:

$$0 = (1-\delta\gamma)\overline{M} - (\overline{M}-1)E(\overline{M^p}) + \delta(\overline{M}-1)\left[E(\overline{M^p}) - (1-\gamma)\epsilon\right]$$

$$0 = \left[1 - \delta\gamma - (1-\delta)E(\overline{M^p}) - \delta(1-\gamma)\epsilon\right]\overline{M} + (1-\delta)E(\overline{M^p}) + \delta(1-\gamma)\epsilon$$

$$\overline{M} = \frac{(1-\delta)E(\overline{M^p}) + \delta(1-\gamma)\epsilon}{(1-\delta)E(\overline{M^p}) + \delta(1-\gamma)\epsilon - (1-\delta\gamma)}$$

$$\text{(B.36)} \qquad \overline{M} = 1 + \frac{(1-\delta\gamma)}{(1-\delta)E(\overline{M^p}) + (\delta - \delta\gamma)\epsilon - (1-\delta\gamma)}.$$

In addition, (B.7) shows that in steady state the price elasticity of demand is $E(\overline{M^p}) = \epsilon + (\epsilon - 1)\gamma\phi(\overline{M^p})$. Using this expression, we rewrite the denominator of the fraction in (B.36) as

$$(1-\delta)\epsilon + (1-\delta)(\epsilon-1)\gamma\phi(\overline{M^p}) + (\delta-\delta\gamma)\epsilon - (1-\delta\gamma) = (\epsilon-1)\left[(1-\delta\gamma) + (1-\delta)\gamma\phi(\overline{M^p})\right].$$

Plugging this result back into (B.36), we obtain the steady-state price markup:

$$\text{(B.37)} \qquad \overline{M} = 1 + \frac{1}{\epsilon-1} \cdot \frac{1}{1 + \frac{(1-\delta)\gamma}{1-\delta\gamma}\phi(\overline{M^p})}.$$

Fifth, we apply the markup-employment relation (B.31) to the steady state to express employment:

$$\text{(B.38)} \qquad \overline{N} = \left[\frac{(\nu-1)\alpha}{\nu} \cdot \frac{1}{\overline{M}}\right]^{1/(1+\eta)}.$$

Sixth, we apply the production function (B.28) to the steady state to express output:

$$\overline{Y} = \overline{A} \cdot \overline{N}^{\alpha}.$$

Proof of Lemma 7. The expression for the steady-state perceived price markup $\overline{M^p}$ comes from (B.34). The expression for the steady-state fairness factor $\overline{F} = F(\overline{M^p})$ follows from combining (15) with (16). Last, the expression for the steady-state elasticity of the fairness function

$$\overline{\phi} = \phi(\overline{M^p}) = -F'(\overline{M^p}) \cdot \frac{\overline{M^p}}{F(\overline{M^p})}$$

comes from noting that with the fairness function (15), $F'(M^p) = -\theta$. The properties that $\overline{M^p}$ and $\overline{\phi}$ are strictly increasing in $\overline{\pi}$, and that $\overline{F}$ is weakly decreasing in $\overline{\pi}$, follow from the assumptions that $\epsilon > 1$, $\gamma \in (0, 1)$, $\theta > 0$, and $1 - \chi \geq 0$.

Proof of Proposition 3. The expressions for the steady-state price markup $\overline{M}$ and steady-state employment $\overline{N}$ come from (B.37) and (B.38). Since $\delta < 1$, $\gamma \in (0, 1)$, and $\overline{\phi} > 0$ is strictly increasing in $\overline{\pi}$ (Lemma 7), it follows that $\overline{M}$ is strictly decreasing in $\overline{\pi}$. And since $\alpha > 0$, $\nu > 1$, $\eta > 0$, and $\overline{M} > 0$ is strictly decreasing in $\overline{\pi}$, it follows that $\overline{N}$ is strictly increasing in $\overline{\pi}$.

Proof of Corollary 2. First, the expressions for the steady-state perceived price markup, $\overline{M^p}$, steady-state fairness factor, $\overline{F}$, and steady-state elasticity of the fairness function, $\overline{\phi}$, in Lemma 7indicate that around the zero-inflation steady state,

$$\overline{M^p} = \frac{\epsilon}{\epsilon - 1}, \qquad \overline{F} = 1, \qquad \overline{\phi} = \frac{\theta\epsilon}{\epsilon - 1}. \tag{B.39}$$

These expressions also show that

$$\begin{aligned}
\frac{d\ln(\overline{M^p})}{d\overline{\pi}} &= \frac{\gamma}{1-\gamma} \\
\frac{d\ln(\overline{F})}{d\overline{\pi}} &= -\theta \cdot (1-\chi) \cdot \frac{\overline{M^p}}{\overline{F}} \cdot \frac{d\ln(\overline{M^p})}{d\overline{\pi}} \\
\frac{d\ln(\overline{\phi})}{d\overline{\pi}} &= \frac{d\ln(\overline{M^p})}{d\overline{\pi}} - \frac{d\ln(\overline{F})}{d\overline{\pi}}.
\end{aligned}$$

Hence, around the zero-inflation steady state, we have

$$\frac{d\ln(\overline{F})}{d\overline{\pi}} = -(1-\chi) \cdot \frac{\theta\epsilon}{\epsilon - 1} \cdot \frac{\gamma}{1-\gamma}$$

and

$$\text{(B.40)} \qquad \frac{d\ln(\overline{\phi})}{d\pi} = \frac{\gamma}{1-\gamma}\left[1+(1-\chi)\cdot\frac{\theta\epsilon}{\epsilon-1}\right].$$

Second, the expression of the steady-state price markup $\overline{M}$ in Proposition 3 yields

$$\text{(B.41)} \qquad \frac{d\ln\left(\overline{M}\right)}{d\ln\left(\overline{\phi}\right)} = \frac{\overline{\phi}}{\overline{M}}\cdot\frac{d\overline{M}}{d\overline{\phi}} = \frac{\overline{\phi}}{\overline{M}}\cdot\frac{1}{\epsilon-1}\cdot\frac{-1}{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right]^2}\cdot\frac{(1-\delta)\gamma}{1-\delta\gamma},$$

and

$$\text{(B.42)} \qquad (\epsilon-1)\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right]\overline{M} = 1+(\epsilon-1)\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right] = \epsilon+(\epsilon-1)\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}.$$

Combining (B.41) and (B.42), we obtain

$$-\frac{d\ln\left(\overline{M}\right)}{d\ln\left(\overline{\phi}\right)} = \frac{(1-\delta)\gamma}{1-\delta\gamma}\cdot\overline{\phi}\cdot\frac{1}{\epsilon+(\epsilon-1)\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}}\cdot\frac{1}{1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}}.$$

Hence, using (B.39), we find that around the zero-inflation steady state,

$$\text{(B.43)} \qquad -\frac{d\ln\left(\overline{M}\right)}{d\ln\left(\overline{\phi}\right)} = \frac{(1-\delta)\gamma}{1-\delta\gamma}\cdot\frac{\theta}{1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta}\cdot\frac{1}{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\epsilon-1}.$$

Third, from the expression of steady-state employment $\overline{N}$ in Proposition 3, we learn that

$$\frac{d\ln(\overline{N})}{d\pi} = \frac{-1}{1+\eta}\cdot\frac{d\ln\left(\overline{M}\right)}{d\ln\left(\overline{\phi}\right)}\cdot\frac{d\ln(\overline{\phi})}{d\pi}.$$

Using (B.40) and (B.43), we therefore find

$$\frac{d\ln(\overline{N})}{d\pi} = \frac{1-\delta}{1+\eta}\cdot\frac{\theta}{1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta}\cdot\frac{1}{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\epsilon-1}\cdot\frac{\gamma^2}{(1-\gamma)(1-\delta\gamma)}\cdot\frac{[1+(1-\chi)\theta]\,\epsilon-1}{\epsilon-1}.$$

Inverting this equation, we obtain the slope of the long-run Phillips curve:

$$\frac{d\pi}{d\ln(\overline{N})} = \frac{1+\eta}{1-\delta} \cdot \frac{(1-\gamma)(1-\delta\gamma)}{\gamma^2} \cdot \frac{\epsilon-1}{\theta} \cdot \frac{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\left[\left(1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right)\epsilon-1\right]}{[1+(1-\chi)\theta]\,\epsilon-1}. \tag{B.44}$$

We now derive comparative statics on the slope of the long-run Phillips curve. We repeatedly use the assumptions that $\delta \in (0,1)$, $\eta > 0$, $\gamma \in (0,1)$, $\theta > 0$, $\epsilon > 1$, and $\chi \in [0,1]$.

First, ϵ influences the slope of the long-run Phillips curve through

$$(\epsilon-1)\frac{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\epsilon-1}{[1+(1-\chi)\theta]\,\epsilon-1},$$

which can be rewritten as

$$\frac{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\epsilon-1}{(1-\chi)\theta\frac{\epsilon}{\epsilon-1}+1}.$$

Since $\epsilon/(\epsilon-1)$ is decreasing in ϵ and

$$\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\epsilon-1$$

is increasing in ϵ, the slope of the long-run Phillips curve is increasing in ϵ.

Second, since χ appears only once in (B.44), it is clear that the slope of the long-run Phillips curve is increasing in χ.

Third, θ influences the slope of the long-run Phillips curve through

$$\frac{1}{\theta} \cdot \frac{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\left[\left(1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right)\epsilon-1\right]}{[1+(1-\chi)\theta]\,\epsilon-1},$$

which can be rewritten as

$$\Xi(\theta) = \frac{\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\left[\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta+\frac{\epsilon-1}{\epsilon}\right]}{\theta\left[(1-\chi)\theta+\frac{\epsilon-1}{\epsilon}\right]}.$$

The function $\Xi(\theta)$ is a quadratic-quadratic rational function, whose behavior can be determined from its asymptotes and zeros. The function has two vertical asymptotes, at

$$\theta = -\frac{\epsilon-1}{(1-\chi)\epsilon} < 0 \quad \text{and} \quad \theta = 0,$$

and a horizontal asymptote, at

$$\Xi = \left[\frac{(1-\delta)\gamma}{1-\delta\gamma}\right]^2 \cdot \frac{1}{1-\chi} > 0. \tag{B.45}$$

Moreover, both zeros of $\Xi(\theta)$ are negative, at

$$\theta = -\frac{1-\delta\gamma}{(1-\delta)\gamma} < 0 \quad \text{and} \quad \theta = -\frac{\epsilon-1}{\epsilon}\cdot\frac{1-\delta\gamma}{(1-\delta)\gamma} < 0.$$

Hence, the function $\Xi(\theta)$ cannot cross the x-axis when $\theta > 0$. This means that it must approach its positive horizontal asymptote from above, decreasing from $+\infty$ when $\theta \to 0^+$ toward the horizontal asymptote (B.45) when $\theta \to +\infty$. Thus, $\Xi(\theta)$ is decreasing in $\theta > 0$, and so is the slope of the long-run Phillips curve.

Fourth, γ influences the slope of the long-run Phillips curve through

$$\frac{(1-\gamma)(1-\delta\gamma)}{\gamma^2}\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\right]\left[\epsilon-1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\theta\epsilon\right],$$

which can be rewritten as

$$\left[\frac{1-\delta\gamma}{\gamma}+(1-\delta)\theta\right]\left[\frac{(\epsilon-1)(1-\gamma)}{\gamma}+\frac{1-\gamma}{1-\delta\gamma}(1-\delta)\theta\epsilon\right].$$

First, $(1-\delta\gamma)/\gamma$ and $(1-\gamma)/\gamma$ are decreasing in γ. Second, since $\delta < 1$, $(1-\gamma)/(1-\delta\gamma)$ is decreasing in γ. Thus, the slope of the long-run Phillips curve is decreasing in γ.

B.4. Log-Linearized Equilibrium

We log-linearize the equilibrium conditions around a steady state. We then use these log-linearized conditions to prove Lemma 6 and Proposition 2. We also use these conditions to compute the impulse responses to monetary and technology shocks that are presented in Figures 1 and 2.

We describe the log-linearized equilibrium through six variables. The first four are the log-deviations from steady state of output, employment, price markup, and perceived price markup: $\widehat{y}(t)$, $\widehat{n}(t)$, $\widehat{m}(t)$, and $\widehat{m^p}(t)$. The final two are the deviations from steady state of the nominal interest rate and inflation rate: $\widehat{i}(t)$ and $\widehat{\pi}(t)$. These six variables are governed by six linear equations.

Log-Linear Equilibrium Conditions. Several equilibrium conditions take a log-linear form, so they can immediately be log-linearized. The first is the monetary-policy rule (10), which implies

$$\widehat{i}(t) = \widehat{i_0}(t) + \psi\widehat{\pi}(t). \tag{B.46}$$

The second is the production function (B.28), which gives

$$\widehat{y}(t) = \widehat{a}(t) + \alpha\widehat{n}(t). \tag{B.47}$$

The third is the markup-employment relation (B.31), which yields

$$\widehat{m}(t) = -(1+\eta)\widehat{n}(t). \tag{B.48}$$

The fourth is the law of motion for the perceived price markup (B.33), which gives

$$\widehat{m^p}(t) = \gamma\left[\widehat{\pi}(t) + \widehat{m^p}(t-1)\right]. \tag{B.49}$$

IS Equation. The fifth equation is the IS equation, which is based on the consumption Euler equation (B.29). We start by computing a log-linear approximation of (B.29), as in Gali (2008, pp. 35–36):

$$\ln(Y(t)) = \mathbb{E}_t(\ln(Y(t+1))) + \mathbb{E}_t(\pi(t+1)) + \rho - i(t),$$

where $\rho = -\ln(\delta)$ is the discount rate. Subtracting the steady-state values of both sides yields

$$\widehat{y}(t) = \mathbb{E}_t(\widehat{y}(t+1)) + \mathbb{E}_t(\widehat{\pi}(t+1)) - \widehat{i}(t).$$

Last, we introduce the values of $\widehat{y}(t)$ and $\widehat{y}(t+1)$ given by (B.47), and the value of $\widehat{i}(t)$ given by (B.46). We obtain the IS equation:

$$\alpha\widehat{n}(t) + \psi\widehat{\pi}(t) = \alpha\,\mathbb{E}_t(\widehat{n}(t+1)) + \mathbb{E}_t(\widehat{\pi}(t+1)) - \widehat{i_0}(t) - \widehat{a}(t) + \mathbb{E}_t(\widehat{a}(t+1)). \tag{B.50}$$

Short-Run Phillips Curve. The sixth and final equation is the short-run Phillips curve. It is based on the pricing equation (B.32).

As a first step toward computing the Phillips curve, we compute the elasticity of the price elasticity of demand $E(M^p) = \epsilon + (\epsilon - 1)\gamma\phi(M^p)$. Given that the elasticity of $\phi(M^p)$ is σ (Lemma 2), the elasticity of $E(M^p)$ at the steady state is

$$\text{(B.51)} \qquad \frac{d\ln(E)}{d\ln(M^p)} = \frac{(\epsilon-1)\gamma\overline{\phi}}{\epsilon+(\epsilon-1)\gamma\overline{\phi}} \cdot \overline{\sigma} \equiv \Omega_0.$$

Second, we introduce the auxiliary function

$$\Lambda_1(M) = \frac{M-1}{M}.$$

The elasticity of $\Lambda_1(M)$ at the steady state is

$$\frac{d\ln(\Lambda_1)}{d\ln(M)} = \frac{\overline{M}}{\overline{M}-1} - 1 = \frac{1}{\overline{M}-1} \equiv \Omega_1.$$

Using the value of $\overline{M}$ in (B.37), we find that Ω_1 satisfies

$$\text{(B.52)} \qquad \Omega_1 = (\epsilon-1)\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right].$$

The left-hand side of (B.32) can be written $LHS = \Lambda_1(M(t)) \cdot E(M^p(t))$. Accordingly, around the steady state the log-linear approximation of LHS is

$$\text{(B.53)} \qquad \ln(LHS) - \ln(\overline{LHS}) = \Omega_1\widehat{m}(t) + \Omega_0\widehat{m^p}(t).$$

Next, we introduce another auxiliary function:

$$\Lambda_2(M^p) = E(M^p) - (1-\gamma)\epsilon = \gamma\left[\epsilon + (\epsilon-1)\phi(M^p)\right].$$

The elasticity of $\Lambda_2(M^p)$ at the steady state is

$$\text{(B.54)} \qquad \frac{d\ln(\Lambda_2)}{d\ln(M^p)} = \frac{(\epsilon-1)\overline{\phi}}{\epsilon+(\epsilon-1)\overline{\phi}} \cdot \overline{\sigma} \equiv \Omega_2.$$

We also introduce the auxiliary function

$$\Lambda_3(x) = 1-\delta\gamma+\delta x,$$

whose elasticity is

$$\frac{d\ln(\Lambda_3)}{d\ln(x)} = \frac{\delta x}{\Lambda_3} \equiv \Omega_3.$$

The right-hand side of (B.32) (abstracting from the expectation operator) can be written

$$RHS = \Lambda_3(\Lambda_1(M(t+1)) \cdot \Lambda_2(M^p(t+1)).$$

Hence, around the steady state the log-linear approximation of RHS is

$$\ln(RHS) - \ln(\overline{RHS}) = \Omega_3 \cdot \left[\Omega_1 \widehat{m}(t+1) + \Omega_2 \widehat{m^p}(t+1)\right], \tag{B.55}$$

where the elasticity Ω_3 is evaluated at $\overline{x} = \overline{\Lambda_1} \cdot \overline{\Lambda_2}$ and $\overline{\Lambda_3} = \overline{RHS} = \overline{LHS} = \overline{E} \cdot \overline{\Lambda_1}$. Thus in (B.55) we have

$$\Omega_3 = \frac{\delta \overline{\Lambda_1} \cdot \overline{\Lambda_2}}{\overline{E} \cdot \overline{\Lambda_1}} = \delta\gamma \frac{\epsilon + (\epsilon - 1)\overline{\phi}}{\epsilon + (\epsilon - 1)\gamma\overline{\phi}}. \tag{B.56}$$

We now bring these results together. Equation (B.32) can be written $LHS = \mathbb{E}_t(RHS)$. This equation also holds in steady state so $\overline{LHS} = \overline{RHS}$. Combining these two equations, we infer

$$\exp\Big(\ln(LHS) - \ln(\overline{LHS})\Big) = \mathbb{E}_t\Big(\exp\Big(\ln(RHS) - \ln(\overline{RHS})\Big)\Big).$$

Around $x = 0$, we have $\exp(x) = 1 + x$. Applying this approximation to both sides of the previous equation, we find

$$1 + \ln(LHS) - \ln(\overline{LHS}) = 1 + \mathbb{E}_t\Big(\ln(RHS) - \ln(\overline{RHS})\Big).$$

We then use the results in (B.53) and (B.55):

$$\Omega_1 \widehat{m}(t) + \Omega_0 \widehat{m^p}(t) = \Omega_3 \cdot \left[\Omega_1 \mathbb{E}_t(\widehat{m}(t+1)) + \Omega_2 \mathbb{E}_t\Big(\widehat{m^p}(t+1)\Big)\right].$$

We divide this equation by Ω_0; insert the values of $\widehat{m}(t)$ and $\widehat{m}(t+1)$ given by (B.48); and insert the value of $\widehat{m^p}(t+1)$ given by (B.49). We obtain

$$-\frac{(1+\eta)\Omega_1}{\Omega_0}\widehat{n}(t) + \widehat{m^p}(t) = -\frac{(1+\eta)\Omega_3\Omega_1}{\Omega_0}\mathbb{E}_t(\widehat{n}(t+1)) + \frac{\gamma\Omega_3\Omega_2}{\Omega_0}\mathbb{E}_t\Big(\widehat{\pi}(t+1) + \widehat{m^p}(t)\Big). \tag{B.57}$$

Using (B.51), (B.52), (B.54), and (B.56), we find that

$$\frac{(1+\eta)\Omega_1}{\Omega_0} = (1+\eta)\frac{\epsilon + (\epsilon - 1)\gamma\overline{\phi}}{\gamma\overline{\phi}\overline{\sigma}}\left[1 + \frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right] \equiv \lambda_1$$

$$\frac{(1+\eta)\Omega_3\Omega_1}{\Omega_0} = (1+\eta)\delta\frac{\epsilon+(\epsilon-1)\overline{\phi}}{\overline{\phi}\overline{\sigma}}\left[1+\frac{(1-\delta)\gamma}{1-\delta\gamma}\overline{\phi}\right] \equiv \lambda_2$$

$$\frac{\gamma\Omega_3\Omega_2}{\Omega_0} = \delta\gamma^2\frac{\epsilon+(\epsilon-1)\overline{\phi}}{\epsilon+(\epsilon-1)\gamma\overline{\phi}}\cdot\frac{(\epsilon-1)\overline{\phi}\overline{\sigma}}{\epsilon+(\epsilon-1)\overline{\phi}}\cdot\frac{\epsilon+(\epsilon-1)\gamma\overline{\phi}}{(\epsilon-1)\gamma\overline{\phi}\overline{\sigma}} = \delta\gamma.$$

Bringing these results into (B.57), we obtain the short-run Phillips curve:

$$(1-\delta\gamma)\widehat{m^p}(t) - \lambda_1\widehat{n}(t) = \delta\gamma\,\mathbb{E}_t(\widehat{\pi}(t+1)) - \lambda_2\,\mathbb{E}_t(\widehat{n}(t+1)). \tag{B.58}$$

Proof of Lemma 6. The law of motion (13) for the perceived price markup comes from (B.49). The expression of the perceived price markup as a discounted sum of past inflation rates is obtained by iterating (B.49) backward; and by noting that $\lim_{T\to\infty}\gamma^T\cdot\widehat{m^p}(t-T)=0$ as $\gamma\in(0,1)$ and $\widehat{m^p}$ is bounded.

Proof of Proposition 2. The short-run Phillips curve (14) comes from (B.58). The hybrid expression of the short-run Phillips curve is obtained by combining (14) with (13).

Blanchard-Kahn Representation. To complete the description of the log-linearized equilibrium, we combine the equilibrium conditions (B.49), (B.50), and (B.58) into a dynamical system of the form proposed by Blanchard and Kahn (1980). Such system is useful to assess the existence and uniqueness of an equilibrium, and to solve for the unique equilibrium when it exists.

We first combine (B.49), (B.50), and (B.58) into a linear dynamical system:

$$\begin{bmatrix}\gamma & \gamma & 0\\ 0 & \psi & \alpha\\ 0 & 0 & \lambda_1\end{bmatrix}\begin{bmatrix}\widehat{m^p}(t-1)\\ \widehat{\pi}(t)\\ \widehat{n}(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & \alpha\\ 1-\delta\gamma & -\delta\gamma & \lambda_2\end{bmatrix}\begin{bmatrix}\widehat{m^p}(t)\\ \mathbb{E}_t(\widehat{\pi}(t+1))\\ \mathbb{E}_t(\widehat{n}(t+1))\end{bmatrix} - \begin{bmatrix}0\\1\\0\end{bmatrix}\omega(t),$$

where

$$\omega(t) = \widehat{i_0}(t) + \widehat{a}(t) - \mathbb{E}_t(\widehat{a}(t+1))$$

is an exogenous shock realized at time t. The inverse of the matrix on the right-hand side is

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & \alpha\\ 1-\delta\gamma & -\delta\gamma & \lambda_2\end{bmatrix}^{-1} = \begin{bmatrix}1 & 0 & 0\\ \frac{(1-\delta\gamma)\alpha}{\lambda_2+\alpha\delta\gamma} & \frac{\lambda_2}{\lambda_2+\alpha\delta\gamma} & \frac{-\alpha}{\lambda_2+\alpha\delta\gamma}\\ \frac{\delta\gamma-1}{\lambda_2+\alpha\delta\gamma} & \frac{\delta\gamma}{\lambda_2+\alpha\delta\gamma} & \frac{1}{\lambda_2+\alpha\delta\gamma}\end{bmatrix}.$$

Premultiplying the dynamical system by the inverse matrix, we obtain the Blanchard-Kahn form of the system:

$$\begin{bmatrix} \widehat{m^p}(t) \\ \mathbb{E}_t(\widehat{\pi}(t+1)) \\ \mathbb{E}_t(\widehat{n}(t+1)) \end{bmatrix} = \begin{bmatrix} \gamma & \gamma & 0 \\ \frac{(1-\delta\gamma)\alpha\gamma}{\lambda_2+\alpha\delta\gamma} & \frac{\lambda_2\psi+\alpha\gamma(1-\delta\gamma)}{\lambda_2+\alpha\delta\gamma} & \frac{(\lambda_2-\lambda_1)\alpha}{\lambda_2+\alpha\delta\gamma} \\ \frac{-(1-\delta\gamma)\gamma}{\lambda_2+\alpha\delta\gamma} & \frac{[\delta\psi+\delta\gamma-1]\gamma}{\lambda_2+\alpha\delta\gamma} & \frac{\lambda_1+\alpha\delta\gamma}{\lambda_2+\alpha\delta\gamma} \end{bmatrix} \begin{bmatrix} \widehat{m^p}(t-1) \\ \widehat{\pi}(t) \\ \widehat{n}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\lambda_2}{\lambda_2+\alpha\delta\gamma} \\ \frac{\delta\gamma}{\lambda_2+\alpha\delta\gamma} \end{bmatrix} \omega(t).$$

This dynamical system determines perceived price markup $\widehat{m^p}(t)$, inflation $\widehat{\pi}(t)$, and employment $\widehat{n}(t)$. All the other variables directly follow.

Under the calibration in Table 3, the Blanchard-Kahn conditions are satisfied, so the equilibrium exists and is determinate. Indeed, under such calibration, the eigenvalues of the matrix in the Blanchard-Kahn system are 0.30, $1.02+0.03i$, and $1.02-0.03i$: one eigenvalue is within the unit circle, and two are outside the unit circle. Further, the dynamical system has one predetermined variable at time t ($\widehat{m^p}(t-1)$) and two nonpredetermined variables ($\widehat{n}(t)$ and $\widehat{\pi}(t)$). As the number of eigenvalues outside the unit circle matches the number of nonpredetermined variables, the solution to the dynamical system exists and is unique (Blanchard and Kahn 1980, Proposition 1).

B.5. Calibration

Finally, we calibrate the fairness-related parameters of the New Keynesian model following the procedure described in Section 5.3. The procedure is based on matching the cost passthroughs estimated in microdata and those obtained by simulating the behavior of a single firm facing a stochastic marginal cost. The calibrated values of the parameters are summarized in Table 3.

Firm Problem. This is a simplified version of the New Keynesian firm problem, which abstracts from hiring decisions. The firm chooses price $P(t)$ and output $Y(t)$ to maximize the expected present-discounted value of profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[P(t) - C(t)\right] Y(t),$$

subject to the demand

$$Y^d(P(t), C^p(t-1)) = P(t)^{-\epsilon} F\left(\left(\frac{\epsilon}{\epsilon-1}\right)^{1-\gamma} \left[\frac{P(t)}{C^p(t-1)}\right]^{\gamma}\right)^{\epsilon-1} \tag{B.59}$$

and to the law of motion (8) for the perceived marginal cost $C^p(t)$. The nominal marginal cost $C(t)$ is exogenous and stochastic.

To solve the firm's problem, we set up the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \Bigg\{ & [P(t) - C(t)]\, Y(t) \\ & + \mathcal{H}(t) \left[Y^d(P(t), C^p(t-1)) - Y(t)\right] \\ & + \mathcal{K}(t) \left[\left[C^p(t-1)\right]^{\gamma} \left[\frac{\epsilon - 1}{\epsilon} P(t)\right]^{1-\gamma} - C^p(t) \right] \Bigg\},\end{aligned}$$

where $\mathcal{H}(t)$ is the Lagrange multiplier on the demand constraint in period t, and $\mathcal{K}(t)$ is the Lagrange multiplier on the perceived marginal cost's law of motion in period t.

First-Order Condition with Respect to Output. The first-order condition with respect to $Y(t)$ is $\partial \mathcal{L} / \partial Y(t) = 0$. It yields

$$\mathcal{H}(t) = P(t) \left[1 - \frac{C(t)}{P(t)}\right],$$

which is the same equation as (B.22) and thus can be rewritten as (B.23).

First-Order Condition with Respect to Price. The first-order condition with respect to $P(t)$ is $\partial \mathcal{L} / \partial P(t) = 0$, which gives

$$0 = Y(t) + \mathcal{H}(t) \frac{\partial Y^d}{\partial P} + (1 - \gamma) \mathcal{K}(t) \frac{C^p(t)}{P(t)}.$$

This equation is the same as (B.24); therefore, it can be re-expressed as (B.25).

First-Order Condition with Respect to Perceived Marginal Cost. Finally, the first-order condition with respect to $C^p(t)$ is $\partial \mathcal{L} / \partial C^p(t) = 0$, which yields

$$0 = \delta\, \mathbb{E}_t \left(\mathcal{H}(t+1) \frac{\partial Y^d}{\partial C^p} + \gamma \mathcal{K}(t+1) \frac{C^p(t+1)}{C^p(t)} \right) - \mathcal{K}(t).$$

Using the elasticity given by (B.8), we get

$$\mathcal{K}(t) = \delta\, \mathbb{E}_t \left(\mathcal{H}(t+1) \frac{Y(t+1)}{C^p(t)} \left[E(M^p(t+1)) - \epsilon\right] + \gamma \mathcal{K}(t+1) \frac{C^p(t+1)}{C^p(t)} \right).$$

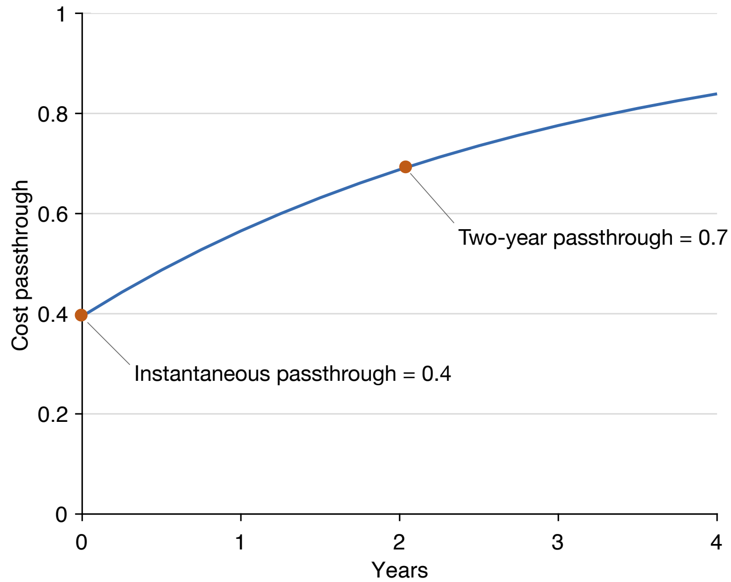

FIGURE B.1. Simulated dynamics of cost passthrough

The cost passthrough represents the percentage increase in price due to a 1% increase in marginal cost. The empirical estimates of the cost passthrough (0.4 and 0.7) are obtained in Section 5.3. The simulations are obtained from the pricing model in Appendix B.5 under the calibration in Table 3.

Next we multiply the equation by $C^p(t)/[Y(t)P(t)]$, and we insert the perceived price markups $M^p(t) = P(t)/C^p(t)$ and $M^p(t+1) = P(t+1)/C^p(t+1)$. We get

$$\frac{\mathcal{K}(t)}{Y(t)M^p(t)} = \delta\,\mathbb{E}_t\left(\frac{Y(t+1)P(t+1)}{Y(t)P(t)}\left\{\frac{\mathcal{H}(t+1)}{P(t+1)}\left[E(M^p(t+1)) - \epsilon\right] + \gamma\frac{\mathcal{K}(t+1)}{Y(t+1)M^p(t+1)}\right\}\right).$$

To conclude, we multiply the equation by $(1-\gamma)$, eliminate $\mathcal{H}(t+1)$ using (B.23), and eliminate $\mathcal{K}(t)$ and $\mathcal{K}(t+1)$ using (B.25). This gives the following pricing equation:

$$\text{(B.60)}\qquad \frac{M(t)-1}{M(t)}E(M^p(t)) = 1 + \delta\,\mathbb{E}_t\left(\frac{Y(t+1)P(t+1)}{Y(t)P(t)}\left\{\frac{M(t+1)-1}{M(t+1)}\left[E(M^p(t+1)) - (1-\gamma)\epsilon\right] - \gamma\right\}\right).$$

In steady state, this equation becomes (B.35) and can therefore be written as (B.37).

Firm Pricing. The firm's pricing is described by four variables: the price $P(t)$, markup $M(t)$, output $Y(t)$, and perceived markup $M^p(t)$. These four variables are determined by four equations: the pricing equation (B.60), the definition of the markup $M(t) = P(t)/C(t)$, the demand curve (B.59), and the perceived markup's law of motion (B.33).

Simulations. We start from a steady-state situation. To be consistent with the simulations of Figures 1 and 2, we assume that steady-state inflation is zero, so the marginal cost C is constant in steady state. Then we impose an unexpected permanent 1% increase in C. We compute the firm's response to this shock by solving the nonlinear dynamical system of four equations that describes firm's pricing. We obtain the dynamics of the cost passthrough by calculating the percentage change in price over time:

$$\beta(t) = \frac{P(t) - \overline{P}}{\overline{P}} \times 100.$$

Calibration Procedure. We set the shape of the fairness function to (7) and the discount factor to $\delta = 0.99$. Then, using the simulations, we calibrate the three main parameters of the model: the concern for fairness, θ, the degree of underinference, γ, and the elasticity of substitution between goods, ϵ. Our goal is to produce an instantaneous cost passthrough of $\beta = 0.4$, a two-year cost passthrough of $\beta = 0.7$, together with a steady-state price markup of $\overline{M} = 1.5$.

Our calibration procedure starts by initializing θ and γ to some values. Using these values and the target $\overline{M} = 1.5$, we compute ϵ from (B.37). In (B.37) we use (A.8), which holds because the fairness function is (7), and because there is no inflation in steady state so customers are acclimated. Using the values of θ, γ, and ϵ, we simulate the dynamics of the cost passthrough.

We repeat the simulation for different values of θ and γ until we obtain a cost passthrough of 0.4 on impact and 0.7 after two years. We reach these targets with $\theta = 9$ and $\gamma = 0.8$. The corresponding value of ϵ is 2.23. The corresponding passthrough dynamics are shown in Figure B.1.

Appendix C. Textbook New Keynesian Model

We describe the textbook New Keynesian model used as benchmark in the simulations of Figures 1 and 2. The model originates in Gali (2008). The pricing friction in the model is the staggered pricing of Calvo (1983). Some researchers alternatively use the price-adjustment cost of Rotemberg (1982). However, since both pricing frictions yield the same linearized Phillips curve around the zero-inflation steady state, simulations in the two cases coincide (Roberts 1995, pp. 976–979).

The model's dynamics around the zero-inflation steady state are governed by an IS equation and a short-run Phillips curve. The IS equation is given by (B.50), as in the model with fairness. This IS equation is obtained from (12) in Gali (2008, Chapter 3), by using logarithmic consumption utility, and by incorporating the monetary-policy rule (B.46) and the production function (B.47).

The short-run Phillips curve is given by

$$\widehat{\pi}(t) = \delta\, \mathbb{E}_t(\widehat{\pi}(t+1)) + \kappa \widehat{n}(t),$$

where

$$\kappa \equiv (1+\eta) \cdot \frac{(1-\xi)(1-\delta\xi)}{\xi} \cdot \frac{\alpha}{\alpha + (1-\alpha)\,\epsilon},$$

and ξ is the fraction of firms keeping their prices unchanged each period. This Phillips curve is obtained from (21) in Gali (2008, Chapter 3), by using logarithmic consumption utility, and by replacing the output gap by $\alpha\widehat{n}(t)$.[1]

The IS equation and short-run Phillips curve jointly determine employment $\widehat{n}(t)$ and inflation $\widehat{\pi}(t)$. The other variables directly follow from $\widehat{n}(t)$ and $\widehat{\pi}(t)$. The nominal interest rate $\widehat{i}(t)$ is given by (B.46). Output $\widehat{y}(t)$ is given by (B.47). The price markup $\widehat{m}(t)$ is given by (B.48). Since households observe both prices and costs, perceived and actual price markups are equal: $\widehat{m^p}(t) = \widehat{m}(t)$.

[1] The output gap is the logarithmic difference between the actual and the natural level of output. The natural levels of output and employment are reached when prices are flexible, so when the price markup is $\epsilon/(\epsilon-1)$. Since $\epsilon/(\epsilon-1)$ is also the steady-state price markup, we infer from (B.31) that the natural and steady-state levels of employment are equal. Hence, (B.28) implies that the natural level of output is $Y^n(t) = A(t)\overline{N}^{\alpha}$. Consequently the output gap is $\ln(Y(t)) - \ln(Y^n(t)) = \alpha[\ln(N(t)) - \ln(\overline{N})] = \alpha\widehat{n}(t)$.

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