Monetary Policy and Unemployment: 
A Matching Approach

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Abstract

This paper analyses the interaction between monetary policy and unemployment using a matching model of the economy. By placing unemployment at the core of the analysis, this matching approach provides insights that complement those obtained with the traditional New Keynesian approach. We model the economy as a single matching market on which labor services are traded. Not all services are sold at all times so sellers of services are unemployed part of the time. Buyers direct their search towards sellers offering lower prices and shorter queues for their services; sellers set their price given this directed search and a price-adjustment cost. Unemployment and inflation are related by a Phillips curve because high unemployment pushes sellers to reduce their prices to attract customers, whereas low unemployment pushes them to increase their prices to take advantage of the long queues of customers. We obtain an optimal monetary policy formula expressed with estimable statistics and thus easy to implement: the gap between the optimal and current intercepts of the interest-rate rule equals the gap between the Hosios and current unemployment rates divided by the response of unemployment to the nominal interest rate. The formula is simple because the Hosios unemployment rate, which minimizes the resources wasted by matching, is also the unemployment rate maintaining inflation at its target level and therefore minimizing resources wasted by price changes. The Hosios unemployment rate is estimable with historical data because it is unaffected by aggregate demand or supply shocks. The formula maintains the unemployment rate at the Hosios level and the inflation rate at its target level.

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1. Introduction

Recessions are costly because of the increase in unemployment that they invariably trigger. As policymakers and academic researchers have recognized this, unemployment has been playing a growing role in the analysis of stabilization policies, in particular monetary policy. The development of New Keynesian models featuring matching frictions on the labor market has been a catalyst of this evolution.\(^1\) This paper proposes a model that goes one step further than existing New Keynesian models by bringing unemployment at the core of the analysis of optimal monetary policy, on par with inflation. By putting more emphasis on unemployment, the model offers a complementary perspective that could enrich our understanding of the interaction between monetary policy and unemployment.

To bring unemployment at the core of the analysis of monetary policy, we propose a model whose structure differs from that of standard New Keynesian models. Our model features a single market for labor services instead of separate markets for goods and labor. As a consequence, there is no need to measure slack on the goods and labor markets separately: unemployment is the sole measure of slack. Furthermore, the model does not distinguish between prices and wages: the price of labor services is both a price and a wage. Thus, the effect of monetary policy on inflation directly impacts the price of labor and thus unemployment.

The economy is composed of self-employed households who sell labor services to other households on a matching market.\(^2\) Unemployment occurs in equilibrium because labor services are traded on a matching market so households are unable to sell their entire productive capacity and part of their capacity is idle at any point in time.

The households consume labor services bought from other households and save part of their income using bonds. When they buy services, households direct their search towards sellers offering lower prices and shorter queues for their services, as in Moen [1997]. When they sell services, households set their price given this directed search and a quadratic price-adjustment cost as in Rotemberg [1982]. Inflation dynamics arise from this new pricing mechanism com-

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\(^{1}\)For a survey of this literature, see Galí [2010].

\(^{2}\)For other matching models incorporating monetary elements, see for instance Lehmann [2012] and Lehmann and Van der Linden [2010].
bining directed search and price-adjustment costs.

The general equilibrium is described by three equations: a consumption Euler equation that describes the consumption-saving decision of households, an interest-rate rule that describes how monetary policy responds to inflation and unemployment, and a Phillips curve relating inflation to unemployment. The mechanism behind the Phillips curve is the following. When unemployment is high, sellers do not sell much, which pushes them to reduce their prices to attract customers, thus generating low inflation. Conversely, when unemployment is low, sellers sell a lot but at a low price, which pushes them to increase their prices and take advantage of the long queues of customers, thus fueling high inflation.

Monetary policy is described by an interest-rate rule that describes how monetary policy responds to inflation and unemployment. The Taylor principle remains valid in our model: when the response to inflation and unemployment is strong enough, the dynamical system describing the equilibrium is a source so that the economy immediately jumps to a new steady state in response to permanent and unexpected shocks.

The goal of the central bank is to adjust the intercept of the interest-rate rule to ensure that the unemployment rate remains at the Hosios [1990] level in response to shocks. The Hosios unemployment rate is the desirable unemployment rate in the model because it minimizes the resources wasted by matching while also maintaining inflation at its target level and therefore minimizing resources wasted by price changes. This property that the Hosios unemployment rate is also the unemployment rate maintaining inflation at its target level is akin to the divine coincidence identified by Blanchard and Galí [2007] in New Keynesian models.

We derive a formula that describes the optimal adjustment of the intercept of the interest-rate rule in response to shocks. The optimal adjustment is particularly simple to implement because it is expressed with estimable statistics: the gap between the optimal and current intercepts of the interest-rate rule equals the gap between the Hosios and current unemployment rates divided by the response of the unemployment rate to the nominal interest rate. Our formula provides a link between the empirics and theory of monetary policy by showing that the response of the unemployment rate to the nominal interest rate—a statistic that has been estimated by a large body of work [Christiano, Eichenbaum and Evans, 1999; Ramey, 2015]—is a key statistic to
design optimal monetary policy.

Another key statistic in the formula is the unemployment gap—the gap between the current unemployment rate and the Hosios unemployment rate. Measuring the unemployment gap necessitates a real-time estimate of the Hosios unemployment rate. It is fairly simple to obtain such an estimate because the Hosios unemployment rate is unaffected by typical business-cycle shocks, either aggregate demand shocks or aggregate supply shocks, so we can use historical labor market data to measure it. Exploiting the methodology developed by Landais, Michaillat and Saez [2010]—this methodology relies on measures of the unemployment rate, the elasticity of the matching function, and the share of the workforce devoted to matching activities—we find that in the United States a reasonable estimate of the Hosios unemployment rate is 4.7%.

In sum, the optimal monetary policy in our model is quite simple. The nominal interest rate should be given by an interest-rate rule. The response of the nominal interest rate to inflation and unemployment should be large enough to ensure that the equilibrium is locally unique. The intercept of the rule should be continuously adjusted to maintain the unemployment rate at the Hosios level—around 4.7% in the United States. The empirical literature suggest that increasing the Federal funds rate by 1 percentage point reduces the unemployment rate by 0.4 percentage points. Hence an increase of the unemployment rate by 1 percentage point should be counteracted with an increase in the intercept of the interest-rate rule by $1/0.4=2.5$ percentage points. The recommended response of monetary policy is substantial and suggests that monetary policy is likely to be constrained by the zero lower bound when the unemployment rate rises above 7%.

2. A Matching Model for the Analysis of Monetary Policy

This section proposes a matching model for the analysis of monetary policy. The economy consists of a measure 1 of identical households and a central bank. Households are self-employed; they produce labor services, consume labor services from other households, and hold bonds. Because we abstract from firms and assume that all production directly takes place within households, the product and labor markets are merged into one single market for labor ser-
The central bank sets an inflation target and the nominal interest rate on bonds. The first point of departure from standard macroeconomic models is that labor services are traded on a matching market, whereas they would usually be traded on a perfectly or monopolistically competitive market. This first assumption introduces unemployment in the model since households are unable to sell all their labor services in equilibrium. The second point of departure is that inflation dynamics arise from a new pricing mechanism combining directed search as in Moen [1997] with costly price adjustment as in Rotemberg [1982]. This second assumption generates a Phillips curve that links inflation to unemployment.

2.1. The Market for Labor Services

Each household has a productive capacity $k$. The productive capacity indicates the maximum amount of services that a household could deliver at any point in time. At time $t$, a household sells $y(t) < k$ services. The price of each unit of labor service is $p(t) > 0$. The rate of inflation at time $t$ is $\pi(t) = \dot{p}(t)/p(t)$, where $p(t)$ is the price level.

All these services are sold through long-term relationships. The idle capacity of the household at time $t$ is $k - y(t)$. Since some of the capacity of the household is idle, some household members are unemployed. The rate of unemployment, defined as the share of workers who are idle, is $u(t) = (k - y(t))/k$, where $k$ is the aggregate productive capacity and $y(t)$ is the aggregate output of services.

Households also consume labor services, but they cannot consume their own services, so they buy services from other households. To purchase labor services, households advertise $v(t)$ vacancies at time $t$. A Cobb-Douglas matching function gives the rate $h(t)$ at which new long-term relationships are formed: $h(t) = \omega \cdot (k - y(t))^\eta \cdot v(t)^{1-\eta}$, where $k - y(t)$ is aggregate idle capacity, $v(t)$ is aggregate number of vacancies, $\omega > 0$ governs the efficacy of matching, and $\eta \in (0, 1)$ is the elasticity of the matching function with respect to available capacity.

The market tightness $x$ is defined by $x(t) = v(t)/(k - y(t))$. The market tightness is the ratio $x(t) = v(t)/(k - y(t))$. The market tightness is the ratio

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3 Michaillat and Saez [2015a] show how the model can be extended to include distinct labor and product markets. In that model, firms hire workers on the labor market and sell their production on the product market.

4 The model has a similar structure to the model developed by Michaillat and Saez [2014], but the present model incorporates inflation dynamics whereas the model in Michaillat and Saez [2014] has fixed inflation.
of the two arguments in the matching function: aggregate vacancies and aggregate idle capacity. Since it is an aggregate variable, households take the market tightness as given. With constant returns to scale in matching, the market tightness determines the rates at which sellers and buyers enter into new long-term trading relationships. At time $t$, each of the $k - y(t)$ available services is committed to a long-term relationship at rate $f(x(t)) = h(t)/(k - y(t)) = \omega \cdot x(t)^{1-\eta}$ and each of the $v(t)$ vacancies is filled with a long-term relationship at rate $q(x(t)) = h(t)/v(t) = \omega \cdot x(t)^{-\eta}$. The function $f$ is increasing and the function $q$ is decreasing. Hence, when the market tightness is higher, it is easier to sell services but harder to buy them.

Long-term relationships separate at rate $s > 0$. Accordingly, output is a state variable with law of motion $\dot{y}(t) = f(x(t)) \cdot (k - y(t)) - s \cdot y(t)$. The term $f(x(t)) \cdot (k - y(t))$ is the number of new relationships formed at $t$. The term $s \cdot y(t)$ is the number of existing relationships separated at $t$. However, in practice, because the transitional dynamics of output are fast, output rapidly adjusts to its steady-state level where market flows are balanced.\(^5\) Throughout the paper, we therefore simplify the analysis by modeling output as a jump variable equal to its steady-state value defined by $f(x(t)) \cdot (k - y(t)) = s \cdot y(t)$.

With this simplification, output becomes a function of market tightness defined by

$$y(x) = \frac{f(x)}{f(x) + s} \cdot k. \tag{1}$$

The function $y(x)$ is in $[0,k]$, increasing on $[0, +\infty)$, with $y(0) = 0$ and $\lim_{x \to +\infty} y(x) = k$. By definition, output is directly related to the unemployment rate: $y = (1 - u) \cdot k$. Hence, the simplification also implies that the unemployment rate is function of market tightness defined by

$$u(x) = \frac{s}{s + f(x)}. \tag{2}$$

The function $u(x)$ is in $[0,1]$, decreasing on $[0, +\infty)$, with $u(0) = 1$ and $\lim_{x \to +\infty} u(x) = 0$. Intuitively, when the market tightness is higher, if it easier to sell services so output is higher and the unemployment rate is lower. The elasticity of $y(x)$ is $(1 - \eta) \cdot u(x)$ and that of $u(x)$ is $-(1 - \eta) \cdot (1 - u(x))$.

\(^5\)Hall [2005], Pissarides [2009], and Shimer [2012] establish this property for the employment rate, which is proportional to output in our model. Michaillat and Saez [2015b] make the same simplifying assumption and provide additional evidence that the transitional dynamics of the employment rate are quantitatively unimportant.
Advertising vacancies is costly. Posting one vacancy costs $\rho > 0$ services per unit time. A total of $\rho \cdot v(t)$ services are spent at time $t$ on filling vacancies. These services represent the resources devoted by households to matching with appropriate providers of services. Since these resources devoted to matching do not enter households’ utility function, we define consumption, denoted $c(t) < y(t)$, as output net of the services used for matching: $c(t) = y(t) - \rho \cdot v(t)$. It is consumption and not output that matters for welfare. This definition of consumption is different from that in national accounts, where $y(t)$ would be called consumption, but defining consumption as output net of recruiting costs is common in the matching literature [for example, Gertler and Trigari, 2009].

As market flows are balanced, $s \cdot y(t) = v(t) \cdot q(x(t))$ and output and consumption satisfy

$$c(t) = y(t) - \rho \cdot \frac{s \cdot y(t)}{q(x(t))}$$

which implies

$$y(t) = \left[1 + \frac{s \cdot \rho}{q(x) - s \cdot \rho}\right] \cdot c(t).$$

Hence, consuming one service requires to purchase $1 + \tau(x)$ services—one service that enters the utility function plus $\tau(x)$ services for matching—where

$$\tau(x) = \frac{s \cdot \rho}{q(x) - s \cdot \rho}.$$  \hfill (3)

From the buyer’s perspective, it is as if it purchased 1 service at a unit price $1 + \tau(x)$, so $\tau(x)$ acts as a wedge on the price of services. The wedge $\tau(x)$ is positive and increasing on $[0, x^m)$, where $x^m \in (0, +\infty)$ is defined by $q(x^m) = \rho \cdot s$. In addition, $\lim_{x \to x^m} \tau(x) = +\infty$. Intuitively, when the market tightness is higher, it is more difficult to match with a seller so the matching wedge is higher. The elasticity of $1 + \tau(x)$ is $\eta \cdot \tau(x)$.

Since $y = (1 + \tau(x)) \cdot c$, we can write consumption as a function of market tightness:

$$c(x) = \frac{1 - u(x)}{1 + \tau(x)} \cdot k.$$  \hfill (4)

The function $c(x)$ plays a central role in the analysis because it gives the amount of services
that can be consumed for a given tightness. The expression (4) shows that consumption is below the capacity $k$ in a matching market because some services are not sold in equilibrium ($u(x) > 0$) and some services are used for matching instead of consumption ($\tau(x) > 0$). The function $c(x)$ is defined on $[0,x^m]$, positive, with $c(0) = 0$ and $c(x^m) = 0$. The elasticity of $c(x)$ is $(1 - \eta) \cdot u(x) - \eta \cdot \tau(x)$. The elasticity is $1 - \eta$ for $x = 0$, $-\infty$ at $x = x^m$, an it is strictly decreasing. Therefore, the function $c(x)$ is strictly increasing for $x \leq x^*$, strictly decreasing for $x \geq x^*$, where the tightness $x^*$ is defined by

$$ (1 - \eta) \cdot u(x^*) = \eta \cdot \tau(x^*). $$(5)

The function $c(x)$ is therefore maximized at $x = x^*$. The tightness $x^*$ is the tightness underlying the condition of Hosios [1990] for efficiency in a matching model. We denote the corresponding unemployment rate by $u^* = u(x^*)$. We refer to $x^*$ and $u^*$ as the **Hosios tightness** and **Hosios unemployment rate**.

Figure 1 summarizes the model. Panel A depicts how output, consumption, and unemployment rate depend on market tightness in feasible allocations. Panel B depicts the functions $c(x)$ and $y(x)$, the Hosios tightness $x^*$, the Hosios unemployment rate $u^*$, situations in which tightness is inefficiently high and unemployment is inefficiently low ($x > x^*, u < u^*$), and situations in which tightness is inefficiently low and unemployment is inefficiently high ($x < x^*, u > u^*$).
When the tightness is inefficiently low, too much of the economy’s productive capacity is idle, and a marginal increase in tightness increases consumption. When the tightness is inefficiently high, too many resources are devoted to purchasing labor services, and a further increase in tightness decreases consumption.

2.2. Monetary Policy

Monetary policy determines the inflation target, $\bar{\pi}$, and the nominal interest rate, $i(t)$.

The inflation target is determined by long-term considerations not captured in the model, so we just take $\bar{\pi}$ as a parameter. In practice the inflation target chosen by central banks (explicitly or implicitly) is positive, around 2%. There are several motivations for choosing a positive inflation target. One possible motivation is that higher average inflation leads to higher average nominal interest rates and thus leaves more room to deal with severe recessions without being constrained by the zero lower bound on nominal interest rates (see the discussion in Coibion, Gorodnichenko and Wieland [2012]).

Households trade nominal bonds on a perfectly competitive market. At time $t$, households hold $b(t)$ bonds, and the rate of return on bonds—the nominal interest rate—is determined by the monetary policy. The interest rate is determined by an interest-rate rule:

$$i(t) = i_0 + \phi_\pi \cdot (\pi(t) - \bar{\pi}) - \phi_u \cdot (u(t) - u^*),$$

where the intercept $i_0 > 0$ gives the nominal interest rate when inflation is at the target $\pi$ and unemployment is at the Hosios level $u^*$, $\phi_\pi > 0$ gives the response of monetary policy to inflation, and $\phi_u > 0$ gives the response of monetary policy to unemployment. Below we will determine the optimal values of $i_0$, $\phi_\pi$, and $\phi_u$ to maximize welfare.
2.3. Households

Households derive utility from consuming labor services. Their instantaneous utility function is \( \ln(c(t)) \). The utility function at time 0 is the discounted sum of instantaneous utilities

\[
\int_0^{+\infty} e^{-\delta t} \cdot \ln(c(t)) \, dt,
\]

where \( \delta > 0 \) is the subjective discount rate. Consuming \( c(t) \) services requires to purchase \((1 + \tau(x(t))) \cdot c(t) \) services and therefore costs \( p(t) \cdot (1 + \tau(x(t))) \cdot c(t) \) where \( p(t) \) is the market price for services and \( x(t) \) is the market tightness.

Besides consuming services provided by other households, each household also sells their own services. Each household is self-employed and sets a price \( P(t) \) for its services.\(^6\) When a seller sets a price, she takes into account the effect of her price on the tightness she faces, which in turn determines how much labor services she sells. Indeed, buyers direct their search towards the most attractive markets: they choose the market where they buy services based on the price \( p \) and the tightness \( x \) in that market. What matters for buyers is the effective price they pay, \( p \cdot (1 + \tau(x)) \). As in Moen [1997], we assume that search for labor services is not random but directed: sellers post their price and buyers arbitrage across sellers until they are indifferent across sellers. For a price \( P \), the tightness that a seller faces is given by

\[
(1 + \tau(X)) \cdot P = e(t)
\]

where \( e(t) \) is the effective price in the economy. The effective price is taken as given by buyers and sellers. This condition says that buyers are indifferent between all sellers. A sellers can choose a high price and face a low tightness thus a low probability to sell. Or she can choose a low price and face a high tightness and thus a high probability to sell.

\(^6\)We denote the price set by the household for its services with a capital letter (\( P \)), and the price paid by the household for others’ services, which is also the market price, with a small letter (\( p \)). We use the same notation to separate between the household’s tightness (\( X \)) and the market tightness (\( x \)), the household’s output (\( Y \)) and the aggregate output (\( y \)), the households capacity (\( K \)) and the market capacity (\( k \)), and between the growth rate of \( P(t) \) and the growth rate of \( p(t) \) (\( \Pi \) versus \( \pi \)). In equilibrium, \( P(t) = p(t), X(t) = x(t), Y(t) = y(t), K(t) = k(t), \) and \( \Pi(t) = \pi(t) \).
Absent any price-adjustment cost, sellers would choose $P$ to maximize $P \cdot (1 - u(X))$ subject to (8). This is equivalent to choosing $X$ to maximize $[1 - u(X)] / [1 + \tau(X)]$. The analysis of Section 2 shows that $[1 - u(x)] / [1 + \tau(x)]$ is maximized at the Hosios tightness $x^\ast$. Therefore, sellers would always set their price such that $X = x^\ast$. This is the efficiency result established by Moen [1997] for matching models with directed search.

However, sellers face price-adjustment costs that prevent them from continuously adjusting their price and maintain their tightness at $x^\ast$. As in Rotemberg [1982], we assume that sellers incur a quadratic cost to adjust prices at a rate that differs from the inflation target, $\pi$. Sellers have a productive capacity of $l$—which means that they can at most sell $l$ services per unit time—but this productive capacity is reduced when sellers have to change prices at a rate different from the inflation target. Specifically, the effective capacity of a seller when they change their prices at time $t$ is $K(t) = (1 - a(\Pi(t))) \cdot l$ where

$$a(\Pi(t)) \equiv \frac{\kappa}{2} \cdot (\Pi(t) - \bar{\pi})^2,$$

$\Pi(t) \equiv (dP(t)/dt)/P(t)$ is the the growth rate of prices for the seller, and $\kappa$ is a parameter governing the cost of changing prices. If $\kappa = 0$, prices adjust at no cost. If $\kappa \to \infty$, it is infinitely costly to change the growth rate of prices from the inflation target. The interpretation for this representation of the price-adjustment cost is that when sellers increase their prices at a rate that is not the publicized inflation target, they devote time to implementing the price adjustment. Customers expect prices to change at the inflation target, so such changes are costless. But customers do not expect other changes, so sellers must devote time to communicating and justifying a price change that does not follow the inflation target. Since this time cannot be used to produce services, sellers have less capacity available during an unusual price change. This representation is motivated by the microevidence collected by Zbaracki et al. [2004]: using time-and-motion methods, they study the pricing process of a large industrial firm; they find that the physical, managerial, and customer costs of changing prices are mostly due to the time required to decide on price changes and communicate and justify them to customers.
If a seller chooses a price \( P \), equation (8) implies that she faces a tightness

\[
X(P) = \tau^{-1}\left(\frac{e}{P} - 1\right).
\]  

A useful result is that the elasticity of \( X(P) \) is \(-1/ (\eta \cdot \tau(x))\).

Households earn income by selling services, spend part of the income on services sold by other households, and save the rest of the income as bonds. The law of motion of a household’s holding of bonds is

\[
\frac{db(t)}{dt} = P(t) \cdot (1 - u(X(P(t)))) \cdot (1 - a(\Pi(t))) \cdot l - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) + i(t) \cdot b(t).
\]  

Here, \( b(t) \) are nominal bond holdings, \( p(t) \) is the market price of services, \( P(t) \) is the price set by the household for its services, \( (1 + \tau(x(t))) \cdot c(t) \) is the quantity of services purchased, and \( (1 - u(X(P(t)))) \cdot (1 - a(\Pi(t))) \cdot l \) is the quantity of services sold. The household is also subject to a typical no-Ponzi condition.

The flow budget constraint is standard but for three elements. First, the capacity of \( l \) is reduced by a factor \( 1 - a(\Pi) \leq 1 \) as sellers devote time to changing prices. Second, the effective capacity of \( (1 - a(\Pi)) \cdot l \) is further reduced by a factor \( 1 - u(X) \leq 1 \) as only a fraction \( 1 - u(X) \) of the capacity is actually sold on the matching market. Third, consumption \( c \) has a price wedge \( 1 + \tau(x) \geq 1 \) because some purchases are used for recruiting such that consuming one service requires buying \( 1 + \tau(x) \) services.

The representative household’s problem is to choose paths for consumption, bond holdings, price, and growth rate of price, \([c(t), b(t), P(t), \Pi(t)]_{t=0}^{+\infty}\) to maximize (7) subject to (10) and to \( dP(t)/dt = \Pi(t) \cdot P(t) \), taking as given the initial bond holdings \( b(0) \), initial price \( P(0) \), and the paths for market tightness, market price, effective price, nominal interest rate, and inflation \([x(t), p(t), e(t), i(t), \pi(t)]_{t=0}^{+\infty}\).
To solve the household’s problem, we set up the current-value Hamiltonian:

\[
\mathcal{H}(t, c(t), b(t), P(t), \Pi(t)) = \ln(c(t))
+ \lambda(t) \cdot [P(t) \cdot (1 - u(X(P(t)))) \cdot (1 - a(\Pi(t))) \cdot l - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) + i(t) \cdot b(t)]
+ \mu(t) \cdot P(t) \cdot \Pi(t)
\]

with control variable \( c(t) \) and \( \Pi(t) \), state variables \( b(t) \) and \( P(t) \), and current-value costate variables \( \lambda(t) \) and \( \mu(t) \). The necessary conditions for an interior solution to this maximization problem are \( \partial \mathcal{H} / \partial c = 0 \), \( \partial \mathcal{H} / \partial \Pi = 0 \), \( \partial \mathcal{H} / \partial b = \delta \cdot \lambda(t) - \dot{\lambda}(t) \), \( \partial \mathcal{H} / \partial P = \delta \cdot \mu(t) - \dot{\mu}(t) \), and appropriate transversality conditions.

The first-order condition \( \partial \mathcal{H} / \partial c = 0 \) yields

\[
1/\lambda(t) = c(t) \cdot p(t) \cdot (1 + \tau(x(t)))
\] (11)

The first-order condition \( \partial \mathcal{H} / \partial b = \delta \cdot \lambda(t) - \dot{\lambda}(t) \) yields

\[
\frac{\dot{\lambda}(t)}{\lambda(t)} = \delta - i(t).
\] (12)

The first-order condition \( \partial \mathcal{H} / \partial \Pi = 0 \) yields

\[
\mu(t) = \lambda(t) \cdot (1 - u(X(P(t)))) \cdot l \cdot \kappa \cdot (\Pi(t) - \pi).
\] (13)

The first-order condition \( \partial \mathcal{H} / \partial P = \delta \cdot \mu(t) - \dot{\mu}(t) \) yields

\[
(\delta - \Pi(t)) \cdot \mu(t) - \dot{\mu}(t) = \lambda(t) \cdot Y(t) - \dot{\lambda}(t) \cdot P(t) \cdot u'(X(P(t))) \cdot X'(P(t)) \cdot (1 - a(\Pi(t))) \cdot l.
\] (14)

Using the derivatives

\[
u'(X) = -(1 - \eta) \cdot (1 - u) \cdot \frac{u}{X}, \quad X'(P) = -\frac{1}{\eta \cdot \tau(X)} \cdot \frac{X}{P},
\]
we simplify this condition to
\[
(\delta - \Pi(t)) \cdot \mu(t) - \dot{\mu}(t) = \lambda(t) \cdot Y(t) - \dot{\lambda}(t) \cdot \frac{1 - \eta}{\eta} \cdot \frac{u(X(P(t)))}{\tau(X(P(t)))} \cdot (1 - u(X(P(t)))) \cdot (1 - a(\Pi(t))) \cdot l \]
\[
(\delta - \Pi(t)) \cdot \mu(t) - \dot{\mu}(t) = \lambda(t) \cdot Y(t) \left[ 1 - \frac{1 - \eta}{\eta} \cdot \frac{u(X(P(t)))}{\tau(X(P(t)))} \right] \]

Since equation (13) implies that
\[
\lambda(t) \cdot Y(t) = \mu(t) \cdot \frac{1 - a(\Pi(t))}{\kappa \cdot (\Pi(t) - \bar{\pi})},
\] the first-order condition becomes
\[
-\frac{\dot{\mu}(t)}{\mu(t)} = \Pi(t) - \delta + \frac{1 - a(\Pi(t))}{\kappa \cdot (\Pi(t) - \bar{\pi})} \left[ 1 - \frac{1 - \eta}{\eta} \cdot \frac{u(X(P(t)))}{\tau(X(P(t)))} \right].
\]

We now differentiate equations (11) and (13) to express $\dot{\lambda}/\lambda$ and $\dot{\mu}/\mu$ has a function of observable variables:
\[
-\frac{\dot{\lambda}(t)}{\lambda(t)} = \pi(t) + \frac{\dot{y}(t)}{y(t)}
\]
\[
-\frac{\dot{\mu}(t)}{\mu(t)} = \frac{\dot{\lambda}(t)}{\lambda(t)} + \frac{\dot{Y}(t)}{Y(t)} + \frac{\Pi(t)}{\Pi(t) - \bar{\pi}} \cdot \frac{1 + a(\Pi(t))}{1 - a(\Pi(t))}.
\]

We obtain the last equation by replacing $(1 - u(X)) \cdot l$ by $Y/(1 - a(\Pi))$ in (13).

2.4. Equilibrium

We define and analyze the equilibrium of the model.

**Definition 1.** A equilibrium consists of paths for tightness, consumption, bond holdings, nominal interest rate, price, and inflation, $[x(t), c(t), b(t), i(t), p(t), \pi(t)]_{t=0}^{\infty}$, such that the following conditions hold: (i) $[c(t), b(t), p(t), \pi(t)]_{t=0}^{\infty}$ solve the representative household’s problem; (ii) monetary policy determines $[i(t)]_{t=0}^{\infty}$; (iii) supply equals demand on the market for bonds; (iv) supply equals demand on the market for labor services.
These equilibrium conditions are standard. The following proposition offers a simple characterization of the equilibrium:

**Proposition 1.** The equilibrium can be described by three variables: inflation \( \pi(t) \), unemployment rate \( u(t) \), and nominal interest rate \( i(t) \). These three variables are described by three equations. The first equation is the consumption Euler equation:

\[
\frac{\dot{u}(t)}{1-u(t)} + \kappa \cdot \frac{\pi(t) - \bar{\pi}}{1 - a(\pi(t))} \cdot \dot{\pi}(t) = \delta + \pi(t) - i(t)
\]  \hspace{1cm} (19)

The second equation is the Phillips curve:

\[
\frac{1 + a(\pi(t))}{1 - a(\pi(t))} \cdot \dot{\pi}(t) = \delta \cdot (\pi(t) - \bar{\pi}) + \frac{1 - a(\pi(t))}{\kappa} \cdot g(u(t)),
\]  \hspace{1cm} (20)

where

\[
g(u) \equiv \frac{1 - \eta}{\eta} \cdot \frac{u}{\tau(u)} - 1
\]

measures the unemployment gap. The third equation is the interest-rate rule, given by (6).

Once \([\pi(t), u(t), i(t)]\) are determined, we can determine \([p(t), x(t), c(t), b(t)]\) using equation (2), \(p(t) = p(0) \cdot \exp\left( \int_0^t \pi(s) ds \right)\), equation (4), and \(b(t) = 0\).

**Proof.** Combining (12) and (17), we obtain

\[
\frac{\dot{y}(t)}{y(t)} = i(t) - \pi(t) - \delta.
\]

Furthermore, since \(y(t) = (1 - u(t)) \cdot (1 - a(\pi(t))) \cdot l\), we have

\[
\frac{\dot{y}(t)}{y(t)} = -\frac{\dot{u}(t)}{1-u(t)} - \kappa \cdot \frac{\pi(t) - \bar{\pi}}{1 - a(\pi(t))} \cdot \pi(t)
\]

Combining these two equations yields the Euler equation.

Combining (18) and (17), we obtain

\[
\frac{\dot{\mu}(t)}{\mu(t)} + \pi(t) = \frac{\pi(t) \cdot 1 + a(\pi(t))}{\pi(t) - \bar{\pi} \cdot 1 - a(\pi(t))}.
\]
Furthermore, equation (16) implies that

$$\frac{\dot{\mu}(t)}{\mu(t)} + \pi(t) = \delta - \frac{1 - a(\pi)}{\kappa \cdot (\pi(t) - \bar{\pi})} \cdot \left[ 1 - \frac{1 - \eta}{\eta} \cdot \frac{u}{\tau(u)} \right].$$

Combining these last two equations yields the Phillips curve.

The consumption Euler equation describes the optimal allocation of the household’s income between consumption and saving. The Phillips curve describes the optimal pricing of the household’s services. The measure of the unemployment gap satisfies $g(u^*) = 0$, $g(u) > 0$ if $u > u^*$, $g(u) < 0$ if $u < u^*$, and

$$g'(u) = \frac{1 + \eta \cdot \tau - (1 - \eta) \cdot u}{\eta \cdot \tau \cdot (1 - u)}.$$

It describes the distance of the unemployment rate from the Hosios unemployment rate, $u^*$.

The Phillips curve, given by (20), shows that the Hosios unemployment rate, $u^*$, is also the unemployment rate maintaining inflation stable at the inflation target, $\bar{\pi}$, so the unemployment rate targets arising from matching and monetary considerations are the same. Indeed, if inflation is constant ($\dot{\pi} = 0$) at the target ($\pi = \bar{\pi}$), the Phillips curve implies that $g(u) = 0$ so the unemployment rate is at the Hosios level.

The result that it is the same rate of unemployment that maximizes consumption for a given inflation rate and that keeps inflation at the inflation target is akin to the divine coincidence identified by Blanchard and Galí [2007] in New Keynesian models. The logic is different, however. When the unemployment rate maximizes consumption for a given inflation rate, reoptimizing the growth rate of prices generates no first-order gain to a seller, so it must have no first-order cost—if the marginal cost and marginal benefit from changing prices were not equal, a seller could change the price and make a profit. This means that when the unemployment rate is $u^*$, the inflation rate is necessarily $\bar{\pi}$. With our pricing mechanism based on directed search and costly price adjustment, it is therefore natural to recover a divine coincidence.

To continue the analysis, we linearize the dynamical system describing the equilibrium around the point with zero inflation and efficient unemployment rate. The following proposition determines the linear dynamic system describing the equilibrium:
**Proposition 2.** Around the point \([\pi, u] = [\pi^*, u^*]\), the equilibrium \([\pi(t), u(t)]\) is described by the following linear dynamical system:

\[
\dot{u}(t) = (1 - u^*) \cdot \left[\delta - i_0 + (1 - \phi_\pi) \cdot (\pi(t) - \overline{\pi}) + \frac{1}{\kappa \cdot \eta \cdot \tau^* \cdot (1 - u^*)} \cdot (u(t) - u^*)\right]
\]

\[
\dot{\pi}(t) = \delta \cdot (\pi(t) - \overline{\pi}) + \frac{1}{\kappa \cdot \eta \cdot \tau^* \cdot (1 - u^*)} \cdot (u(t) - u^*)
\]

If \(i_0 = \delta\), the system simplifies to

\[
\dot{u}(t) = (1 - u^*) \cdot \left[(1 - \phi_\pi) \cdot (\pi(t) - \overline{\pi}) + \phi_u \cdot (u(t) - u^*)\right]
\]

\[
\dot{\pi}(t) = \delta \cdot (\pi(t) - \overline{\pi}) + \frac{1}{\kappa \cdot \eta \cdot \tau^*} \cdot (u(t) - u^*)
\]

**Proof.** We linearize (19) and (20) around \([\pi, u] = [\pi^*, u^*]\), and substitute the interest-rate rule into (21). We use \(a(0) = 0\), the derivative of \(g(u)\) obtained above, and \(\dot{\pi} = 0\) at \([\pi, u] = [\overline{\pi}, u^*]\).

The first equation is obtained by combining the linearized consumption Euler equation with the interest-rate rule. The second equation is the linearized Phillips curve. The linear system is similar to the dynamical system describing the equilibrium in a linearized continuous-time New Keynesian model [for example, Werning, 2012].

Proposition 3 establishes the properties of the dynamical system obtained in Proposition 2:

**Proposition 3.** The linear dynamical system (21)–(22) describes the equilibrium value of the variables \([u(t), \pi(t)]\). There are three possible steady-state equilibria:

- If \(i_0 = \delta\), the inflation rate is at the target level and the unemployment rate is at the Hosios level: \(\pi = \overline{\pi}\) and \(u = u^*\).

- If \(i_0 > \delta\), the inflation rate is below the target level and the unemployment rate is above the Hosios level: \(\pi < \overline{\pi}\) and \(u > u^*\).

- If \(i_0 < \delta\), the inflation rate is above the target level and the unemployment rate is below the Hosios level: \(\pi > \overline{\pi}\) and \(u < u^*\).
Since the system has two jump variables and no state variable, the equilibrium is locally unique if and only if the dynamical system is a source around the steady state. In that case, the equilibrium jumps from one steady state to another in response to small unexpected permanent shocks. This happens if and only if

\[ \delta \cdot [(1 - u^*) \cdot \phi_u + i_0 - \delta] + \frac{1}{\kappa \cdot \eta \cdot \tau^*} \cdot (\phi_\pi - 1) > 0. \] (25)

In the case with \( i_0 = \delta \) and \( \phi_u = 0 \), the system is a source if and only if \( \phi_\pi > 1 \).

**Proof.** To ease notation, let \( m_0 \equiv (1 - u^*) \cdot (i_0 - \delta) \), \( m_1 \equiv (1 - u^*) \cdot (\phi_\pi - \delta) \), \( m_2 \equiv (1 - u^*) \cdot \phi_u + i_0 - \delta \), \( m_3 \equiv \delta \), and \( m_4 \equiv 1/[(\kappa \cdot \eta \cdot \tau^* \cdot (1 - u^*))]. \)

The steady-state equilibrium is obtained by setting \( \dot{u} = \dot{\pi} = 0 \) in (21)–(22). We find that the steady-state equilibrium is given by

\[ \pi - \pi^* = \frac{-m_4}{m_3} \cdot (u - u^*) \]
\[ u - u^* = \frac{m_0}{m_2 + m_4 \cdot m_1/m_3}. \]

Hence \( u > u^* \) and \( \pi < \pi^* \) iff \( m_0 > 0 \) or \( i_0 > \delta \).

The matrix describing the linear system (21)–(22) is

\[
M = \begin{bmatrix}
m_2 & -m_1 \\
m_4 & m_3
\end{bmatrix}.
\]

The trace of the matrix is \( m_2 + m_3 = i_0 + (1 - u^*) \cdot \phi_u > 0 \). The determinant of the matrix is positive iff (25) holds. Under this condition both trace and determinant of the matrix are positive so the dynamical system is a source.\(^7\)

The phase portrait of the system (21)–(22) is presented in Figure 2. It is fairly similar to the phase portrait obtained in simple New Keynesian models.

---

\(^7\)If the eigenvalues of the matrix are real, the fact that the trace and determinant of the matrix are positive implies that the two eigenvalues are positive, which in turn implies that the system is a source. If the eigenvalues of the matrix are complex conjugates, the fact that the trace of the matrix is positive implies that the real part of the two eigenvalues is positive, which also implies that the system is a source.
The proposition is equivalent to the Taylor principle in New Keynesian models: an interest-rate rule that responds strongly enough to the inflation rate and unemployment rate ensures that the equilibrium is locally unique. In that case the equilibrium jumps directly to steady state in response to small shocks: there are no transitional dynamics. Hence, business cycles are sequences of steady states at different levels of unemployment and inflation following sequences of permanent unexpected shocks.

### 3. The Optimal Response of Monetary Policy to Unemployment Fluctuations

This section studies optimal monetary policy. It argues that an interest-rate rule responding to inflation and unemployment can perfectly stabilize the economy as long as the intercept of the rule is chosen correctly. It then develops a formula to set the intercept at its optimal level using estimable statistics. Because the formula is based on estimable statistics, it could be implemented in practice. Throughout the discussion of optimal monetary policy we abstract from the zero lower bound.  

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8Studying the zero lower bound is outside of the scope of this paper. As in New Keynesian models, the analysis of the zero lower bound raises interesting but difficult issues [for example, Cochrane, 2015; Werning, 2012].
3.1. An Optimal Monetary-Policy Formula

Monetary policy determines the coefficients $\phi_\pi$ and $\phi_u$ of the interest-rate rule. It is optimal to choose the coefficients such that the equilibrium is locally determinate. Thus $\phi_\pi$ and $\phi_u$ must satisfy condition (25).

As monetary policy ensures that the equilibrium is locally determinate, the equilibrium is always at steady state. The steady-state equilibrium can be parametrized by $i_0$, the intercept of the interest-rate rule chosen by the central bank. Hence, we can denote by $u(i_0)$ and $\pi(i_0)$ the equilibrium unemployment rate and inflation rate when the intercept is $i_0$. The central bank chooses $i_0$ to maximize welfare.

Maximizing welfare requires to set $i_0$ such that the inflation rate is at the target level $\pi$ and the unemployment rate is at the Hosios level $u^*$. In that case, no resources are dissipated to change prices and the amount of resources wasted by matching is minimized, so consumption and thus welfare is maximized. As showed by Proposition 2, this requires to set $i_0 = \delta$.

The discount rate $\delta$ is not observable, however, so it is impractical to set $i_0$ equal to $\delta$. Instead, we propose a formula expressed only with estimable statistics that shows how $i_0$ should be adjusted in response to shocks. Effectively this formula ensures that $i_0 = \delta$, but it does so using observable variables and statistics. The following proposition establishes the formula:

**Proposition 4.** Optimal monetary policy targets the Hosios unemployment rate: it sets $i_0$ such that $u(i_0) = u^*$. The Phillips curve (24) shows that as a result, inflation hits the inflation target: $\pi(i_0) = \pi$.

Starting from an initial equilibrium $[i_0, u(i_0)]$, the optimal interest rate $i_0^*$ is such that

$$i_0^* - i_0 \approx \frac{u^* - u(i_0)}{du/di_0},$$

(26)

where the approximation is valid up to a remainder that is second order. The statistic $du/di_0 > 0$ is the effect of $i_0$ on $u$: it measures the change of the unemployment rate (in percentage point) when the interest rate changes by 1 percentage point for exogenous reasons. If $i_0^* < 0$, the optimal monetary policy is constrained by the zero lower bound.
Figure 3: The Fluctuations in Inflation and Unemployment in the United States, 1994–2014

Notes: The unemployment rate is the quarterly average of the seasonally adjusted monthly civilian unemployment rate constructed by the Bureau of Labor Statistics from the Current Population Survey. The rate of core inflation is the (annualized) growth rate of the quarterly average of the seasonally adjusted monthly personal consumption expenditures index (excluding food and energy) constructed by the Bureau of Economic Analysis as part of the National Income and Product Accounts. The shaded areas represent the recessions identified by the NBER.

Proof. The nominal interest rate needs to be set so that \( u(i_0^*) = u^* \). A first-order expansion of \( u(i_0) \) around \( i_0^* \) gives

\[
u(i_0) - u^* = (i_0 - i_0^*) \cdot \frac{du}{di_0} + O((i_0 - i_0^*)^2).
\]

Rearranging the terms, we obtain the approximation in (26).

Optimal monetary policy simply depends on two statistics: (1) the departure of the unemployment rate from the Hosios unemployment rate, (2) the effect of an exogenous change in nominal interest rate on the unemployment rate. Note that \( \frac{du}{di_0} \) can be evaluated either at \( i_0^* \) or at \( i_0 \), as these derivatives are equal up to a first-order approximation. Below we discuss how these two statistics can be estimated.

In Proposition 4, we use the unemployment rate as a target for monetary policy: we show that it is optimal for monetary policy to bring the unemployment rate to the Hosios level, \( u^* \). Given the equilibrium relationships between all the variables, however, it would have been imaginable to use other variables as target—for instance, the inflation rate or output. We chose the unemployment rate as a target because it has two advantages. First, the unemployment target (the Hosios unemployment rate) is stable over time as it does not respond to aggregate demand and supply shocks. This property is visible in equation (5): the Hosios tightness and the Hosios unemployment rate solely depend on the matching function (\( \eta, \omega \)), the matching cost (\( \rho \)), and the separation rate (\( s \)); therefore, it does not respond to aggregate supply shocks (\( l \)) or aggregate

21
demand shocks ($\delta$). Since the theory predicts that $u^*$ is stable over time, we will use historical data to estimate it. Second, the unemployment rate is subject to wide fluctuations over time, which makes it easy to measure deviations of the unemployment rate from its target.

Other candidate targets for the conduct of monetary policy do not possess the qualities of the unemployment rate. Output would not be a good variable to target because the output target is not stable over time. The output target is the output in the welfare-maximizing steady state: $y^* = (1 - u^*) \cdot l$. Aggregate supply shocks are shocks to the capacity $l$; they lead to changes in $y^*$. This means that $y^*$ moves with aggregate supply shocks and cannot be estimated from historical data. This fact that it is difficult to measure the output gap in real time is well known [Galí, 2008]. While the inflation target is stable over time (it is set by long-run monetary policy), it is very hard to measure deviations of inflation from the target because the fluctuations in inflation are small and noisy, as illustrated in Figure 3. A formula based on the inflation gap would therefore generate a very noisy recommendation for the optimal interest rate. It is because the target output moves and actual inflation is noisy that we prefer to use the unemployment gap in our optimal monetary policy formula.

Because the formula is simple and based on estimable statistics, it should be relatively easy to implement it in practice [Chetty, 2009; Galí, 2008]. Our formula describes how the intercept of the interest-rate rule used by the central bank should be adjusted in response to shocks. Once the intercept is adjusted according to the formula, the unemployment rate returns to the Hosios level and the inflation rate to the target level. Accordingly, the slopes of the interest-rate rule are unimportant: they should just be large enough to ensure that the equilibrium is locally unique.

This approach differs from that followed in the New Keynesian literature. In that literature it is usually considered that the intercept of the interest-rate rule cannot be adjusted in real time, so researchers devise interest-rate rules with fixed intercepts but optimized slopes to approximate the optimal policy as closely as possible [Galí, 2008; Taylor and Williams, 2010]. The quality of the approximation offered by the simple rules in New Keynesian models can only be established using simulations; therefore, the level of quality only applies to a specific model for a specific calibration.
Table 1: Effect of the Federal Funds Rate on the Unemployment Rate

<table>
<thead>
<tr>
<th></th>
<th>$du/di$</th>
<th>Method</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernanke and Blinder [1992]</td>
<td>0.6</td>
<td>VAR</td>
<td>1959–1989</td>
</tr>
<tr>
<td>Leeper, Sims and Zha [1996]</td>
<td>0.1</td>
<td>VAR</td>
<td>1960–1996</td>
</tr>
<tr>
<td>Christiano, Eichenbaum and Evans [1996]</td>
<td>0.1</td>
<td>VAR</td>
<td>1965–1995</td>
</tr>
<tr>
<td>Bernanke, Boivin and Eliasz [2005]</td>
<td>0.2</td>
<td>FAVAR</td>
<td>1959–2001</td>
</tr>
<tr>
<td>Coibion [2012]</td>
<td>0.5</td>
<td>narrative</td>
<td>1970–1996</td>
</tr>
</tbody>
</table>

Notes: The statistic $du/di$ measures the change in unemployment rate (in percentage points) when the Federal funds rate changes by 1 percentage point.

3.2. Estimating the Statistics in the Formula

Implementing formula (26) requires to estimate the effect of monetary policy on unemployment, $du/di_0$, and the Hosios unemployment rate, $u^*$. This subsection provides estimates of $du/di_0$ and $u^*$.

Using different techniques, a large literature has estimated the effect of the Federal funds rate on unemployment but also output and other aggregate variables. Because, the central bank choice of $i$ responds to economic shocks, identifying the effect of $i$ on unemployment or output is challenging. The literature has developed VAR empirical models to isolate exogenous shocks to $i$—departures of $i$ from expected responses from the central bank—and Romer and Romer [2004] pioneered a narrative approach to isolate these shocks. Christiano, Eichenbaum and Evans [1999] summarize the qualitative findings of the literature as follows: “The nature of this agreement is as follows: after a contractionary monetary policy shock, short term interest rates rise, aggregate output, employment, profits and various monetary aggregates fall, the aggregate price level responds very slowly, and various measures of wages fall, albeit by very modest amounts.” Table 1 summarizes the estimates in some representative studies. These estimates are in the 0.1–0.9 range, and the mean is 0.4. We could take 0.4 as a mid-range estimate for $du/di$.

Measuring the unemployment gap $u − u^*$ requires measuring the Hosios unemployment rate

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9Christiano, Eichenbaum and Evans [1999], Boivin, Kiley and Mishkin [2010], and Ramey [2015] provide comprehensive surveys of this large empirical literature.
We measure $u^*$ using the definition (5), which can be rewritten

$$\frac{\tau(u^*)}{u^*} = \frac{1 - \eta}{\eta}. \tag{27}$$

As we have argued above, the Hosios unemployment rate depends neither on aggregate demand ($\delta$) nor on aggregate supply ($l$) so we can expect it to be stable over time under the presumption that aggregate demand and supply shocks are the main drivers of business-cycle fluctuations. Moreover, the Hosios unemployment rate only depends on the matching cost ($\rho$), the separation rate ($s$), and the matching function ($\eta$ and $\omega$)—it depends on the parameters $\rho$, $s$, and $\omega$ because they all enter the function $\tau$. The empirical literature estimating the matching function finds that it is very stable [Petrongolo and Pissarides, 2001]; this result suggests that $\omega$ and $\eta$ are stable over time. Analyzing US labor market data, Hall [2005] and Shimer [2012] show that the fluctuations of the separation rate are small at business-cycle frequency, especially since the 1980s; this result suggests that $s$ is stable over time. There is little evidence on the size of the matching cost over time, but we could expect it to be stable. With all these parameters being stable over time, it seems likely that the Hosios unemployment rate is stable over time. We can therefore use the unemployment rate measured in the months when (27) was satisfied to learn about the current value of the Hosios unemployment rate.

Since the Hosios unemployment rate is unaffected by typical business-cycle shocks, either aggregate demand or supply shocks, we can use historical labor market data and do not require real-time data to measure the Hosios unemployment rate. We compare the time series for $\tau/u$ with the constant $(1 - \eta)/\eta$; the unemployment rate when these two series are equal is the the Hosios unemployment rate. First, we measure $u$ with the unemployment rate constructed by the BLS from the CPS. Second, we set $\eta = 0.66$ using the estimate from Landais, Michaillat and Saez [2010]. This estimate is in line with the estimates found in the vast empirical literature describing the matching function: in their survey, Petrongolo and Pissarides [2001] conclude that estimates of $\eta$ fall between 0.5 and 0.7.

The last, and most difficult step, is to compute $\tau$—the ratio of services devoted to matching to services devoted to consumption. We use the series for $\tau$ constructed by Landais, Michaillat and Saez [2010] for the United States, 1990–2014. The series for $\tau$ is displayed in Figure 4.
Panel A. This series is a composite of three underlying series constructed using data from the Bureau of Labor Statistics. The three series measure in different ways the share of workers devoted to matching in the US workforce. The first underlying series is based on the size of the recruiting industry in the United States. The recruiting industry is the industry with North American Industry Classification System (NAICS) code 56131 and official name is “employment placement agencies and executive search services”. It comprises firms primarily engaged in listing employment vacancies and referring or placing applicants for employment, and firms providing executive search, recruitment, and placement services. The number of workers in the recruiting industry is computed by the BLS from the Current Employment Statistics (CES) survey. The two other underlying series are computed using (3) and measures of \( s \) and \( q(x) \) coming from the data computed by the BLS from the Current Population Survey (CPS) and Job Openings and Labor Turnover Survey (JOLTS). In the three cases, the series are scaled such that the share of labor devoted to recruiting in 1997 is 2.5%, thus matching the evidence from the 1997 National Employer Survey (NES). This survey, conducted by the Census Bureau, gathered data from 4500 establishments on their methods for recruiting applicants. Firms in the survey reported spending 2.5% of their labor costs in recruiting activities [Villena Roldan, 2010].

Figure 4. Panel B, compares \( \tau / u \) with \( (1 − \eta)/\eta \) in the United States. Because \( \tau \) is
procyclical and $u$ is sharply countercyclical, $\tau/u$ is very procyclical: it is as high as 0.74 in booms (in 2000:Q2) and as low as 0.16 in slumps (in 2009:Q4). The series $\tau/u$ crosses $(1 - \eta)/\eta = 0.66/0.34 = 0.5$ on several occasions: in 1997:Q3, when the unemployment rate is 4.9%; in 2001:Q3, when the unemployment rate is 4.8%; in 2006:Q1, when the unemployment rate is 4.7%; in 2006:Q3, when the unemployment rate is 4.6%; and in 2007:Q3, when the unemployment rate is 4.7%. The fact that the unemployment rate is very similar in the 5 instances where condition (27) is satisfied is a further indication that the Hosios unemployment rate is stable over time. Averaging these values, we could take $u^* = 4.7\%$ as a mid-range estimate of the Hosios unemployment rate. This value is lower than the estimates (short-term and long-term) of the natural rate of unemployment computed by the US Congressional Budget Office in the 1990–2014 period: their estimate fluctuated between 5% and 6%. To our knowledge, this is the first microfounded estimate of the desirable rate of unemployment in the United States.

3.3. Applying the Formula

In practice it seems that the Federal Reserve reduces the federal funds rate when they are faced with a sudden increase an unemployment, as recommended by formula (26). Indeed, the behavior of the Federal Reserve since the early 1990s appears roughly consistent with our formula for an estimate of $du/di = 0.4$. With $du/di = 0.4$, correcting an unemployment gap of 1 percentage point requires changing $i_0$ by $1/0.4 = 2.5$ percentage points.

The effect of $i_0$ takes a bit of time: empirical estimates suggest that the response of most variables to a monetary policy shock takes several quarters, and the response does not reach its maximum until 4–6 quarters [Christiano, Eichenbaum and Evans, 1999]. On the other hand, increases in unemployment during downturns tend to happen quickly, within a few quarters. The implication is that monetary policy has probably not had much traction when unemployment peaks. Hence, as a first approximation, the initial shock $u - u^*$ can be estimated using the peak value reached by $u$ during a downturn and $u^* = 4.7\%$. We can then observe by how much the Federal Reserve lowers $i$ to counteract the downturn, a change denoted by $i - i^*$ (the interest rate before the downturn minus the new interest rate). If we assume that the interest-rate rule does not respond to unemployment ($\phi_u = 0$) and if we abstract from the small fluctuations in
inflation observed since 1990 (consistent with a flat Phillips curve), then the observed change \( i - i^* \) is very close to the change \( u_0 - u_0^* \). By comparing \( u - u^* \) with \( i - i^* \), we can therefore infer the value of \( du/di \) that rationalizes the behavior of the Federal Reserve.

We focus on the three downturns since 1990, described in Figure 5. For the 1991 downturn, \( u \) went up from 5.2% to 7.8% while \( i \) declined from 8.3% to 2.9%. So for \( u - u^* = 3.1\% \), the Federal Reserve reduced the nominal interest rate by \( i - i^* = 5.4\% \), consistent with \( du/di = 3.1/5.4 = 0.6 \). For the 2001 downturn, \( u \) went up from 3.9% to 5.7% while \( i \) declined from 6.5% to 1.8%. So for \( u - u^* = 1\% \), the Federal Reserve reduced the nominal interest rate by \( i - i^* = 4.7\% \), consistent with \( du/di = 1/4.7 = 0.2 \). In the Great recession, \( u \) went up from 4.6% to 10% and \( i \) went down from 5% all the way to the zero lower bound. So for \( u - u^* = 5.3\% \), the Federal Reserve would have liked to reduce the nominal interest rate by more than 5%, consistent with \( du/di < 5.3/5 = 1.1 \). If peak unemployment is higher than what we consider, because monetary policy has effects before the peak is reached, then the effect of \( i \) on \( u \) revealed by the Federal Reserve’s behavior could be larger.

Overall, the behavior of the Federal Reserve seems roughly consistent with the mid-range empirical estimate of \( du/di = 0.4 \). Large estimates for \( du/di \) (say \( du/di > 1 \)) are difficult to rationalize with the observed behavior of the Federal Reserve, short of believing that downturns
with no response of monetary policy would be extremely severe and monetary policy becomes effective very quickly. Smaller estimates (say $du/di \approx 0$) are logically possible as the Federal Reserve might erroneously believe that $i$ affects the economy when in reality it does not.

At the zero lower bound, monetary policy cannot accommodate shocks because the nominal interest rate is constrained to remain at zero. With a nominal interest rate in good times around 5 percent, this means that the largest shock monetary policy can accommodate is $u - u^* = 0.05 \cdot du/di \approx 0.02$ for an estimate $du/di = 0.4$. Hence any unemployment shock that brings the unemployment rate above $4.7\% + 2\% = 6.7\%$ could bring the economy to the zero lower bound. Such an unemployment shock is not very large, so it is perhaps not surprising that zero-lower-bound episodes have happened. In this context, it seems important to explore other forms of stabilization that could supplement monetary policy in severe downturns—government purchases, tax policies, or macroprudential policies.\footnote{A growing literature is focusing on this type of questions. For recent results, see for instance Eggertsson and Woodford [2006], Mankiw and Weinzierl [2011], Woodford [2011], Werning [2012], Correia et al. [2013], Farhi and Werning [2013], Korinek and Simsek [2016], and Michaillat and Saez [2015b].}

\section{Conclusion}

This paper develops a matching model to analyze the interaction between monetary policy and unemployment. The economy consists of a single matching market on which labor services are traded. Not all services are sold at all times so sellers of services are unemployed part of the time. Buyers direct their search towards sellers offering lower prices and shorter queues for their services, as in Moen [1997]. Sellers set their price given this directed search and a quadratic price-adjustment cost as in Rotemberg [1982]. This new pricing mechanism combining directed search and costly price adjustments generates a Phillips curve that relates unemployment and inflation: high unemployment pushes sellers to reduce their prices to attract customers and leads to low inflation; conversely, low unemployment pushes sellers to increase their prices to take advantage of the long queues of customers and leads to high inflation.

We derive several results that reinforce some of the results obtained in the New Keynesian literature. First, we find that the unemployment rate minimizing the resources wasted because of matching (the unemployment rate described by Hosios [1990]) is the same as the unemployment
rate maintaining inflation stable at its target level and therefore minimizing the resources wasted because of the price-adjustment costs. This property is akin to the divine coincidence identified by Blanchard and Galí [2007] in New Keynesian models. It implies that the optimal monetary policy maintains the unemployment rate at the Hosios level and the inflation rate at its target level. (The inflation target is set by long-term monetary policy based on considerations not modeled here.) Second, we find that monetary policy needs to respond strongly enough to inflation and unemployment to ensure that the equilibrium of the model is locally unique. This property is akin to the Taylor principle in New Keynesian models [Galí, 2008, chapter 4]. The divine coincidence and Taylor principle are therefore robust properties: they extend beyond the monopolistic framework in which they were first derived and are also valid in a matching framework.

We also obtain new results that could enrich our understanding of the optimal response of monetary policy to unemployment fluctuations. Principally, we propose a formula that uses available empirical estimates to describe how the intercept of the interest-rate rule used by the central bank should be adjusted in real time in response to unemployment shocks. We find that the gap between the optimal and current intercepts of the interest-rate rule equals the gap between the Hosios and current unemployment rates divided by the response of unemployment to the nominal interest rate. The formula ensures that the unemployment rate remains at the Hosios level and the inflation rate at its target level.

Estimates of the statistics in our optimal monetary-policy formula are relatively easy to obtain. There is a large literature estimating the response of the unemployment rate to the nominal interest rate; our formula can leverage these existing estimates. To measure the Hosios unemployment rate, we use the methodology developed by Landais, Michaillat and Saez [2010]. This unemployment rate can be measured with historical data because it is unaffected by typical business-cycle shocks—aggregate demand shocks or aggregate supply shocks. We find that in the United States a reasonable estimate of the Hosios unemployment rate is 4.7%. This possibly is the first microfounded estimate of the desirable rate of unemployment in the United States—existing estimates of the “natural rate of unemployment” are usually obtained using trends and other statistical techniques.
Our simple formula allows the central bank to implement an optimal interest-rate rule. In contrast, the simple interest-rate rules used designed with New Keynesian models are only approximations of the optimal policy valid for the specific range of calibrations and shocks considered [Galí, 2008; Taylor and Williams, 2010]. Compared to the New Keynesian literature, our formula also shifts the focus from the slope of the interest-rate rule to its intercept.

References


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