LECTURE 27
SOLOW | GOLDEN RULE

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THE GOLDEN RULE

- different saving rates imply different steady states
- which is the best saving rate and steady state?
- what matters is not how much is produced but how much is consumed
  - consumption = production – investment
- the golden rule describes the best saving rate (and best capital per worker):
  - they maximize consumption per worker

GOLDEN-RULE STEADY STATE

- the golden-rule saving rate achieves the best steady state

- the best steady state has the highest possible consumption per person:

  - $c^* = y^* - i^* = y^* - s \times y^* = (1 - s) \times f(k^*)$

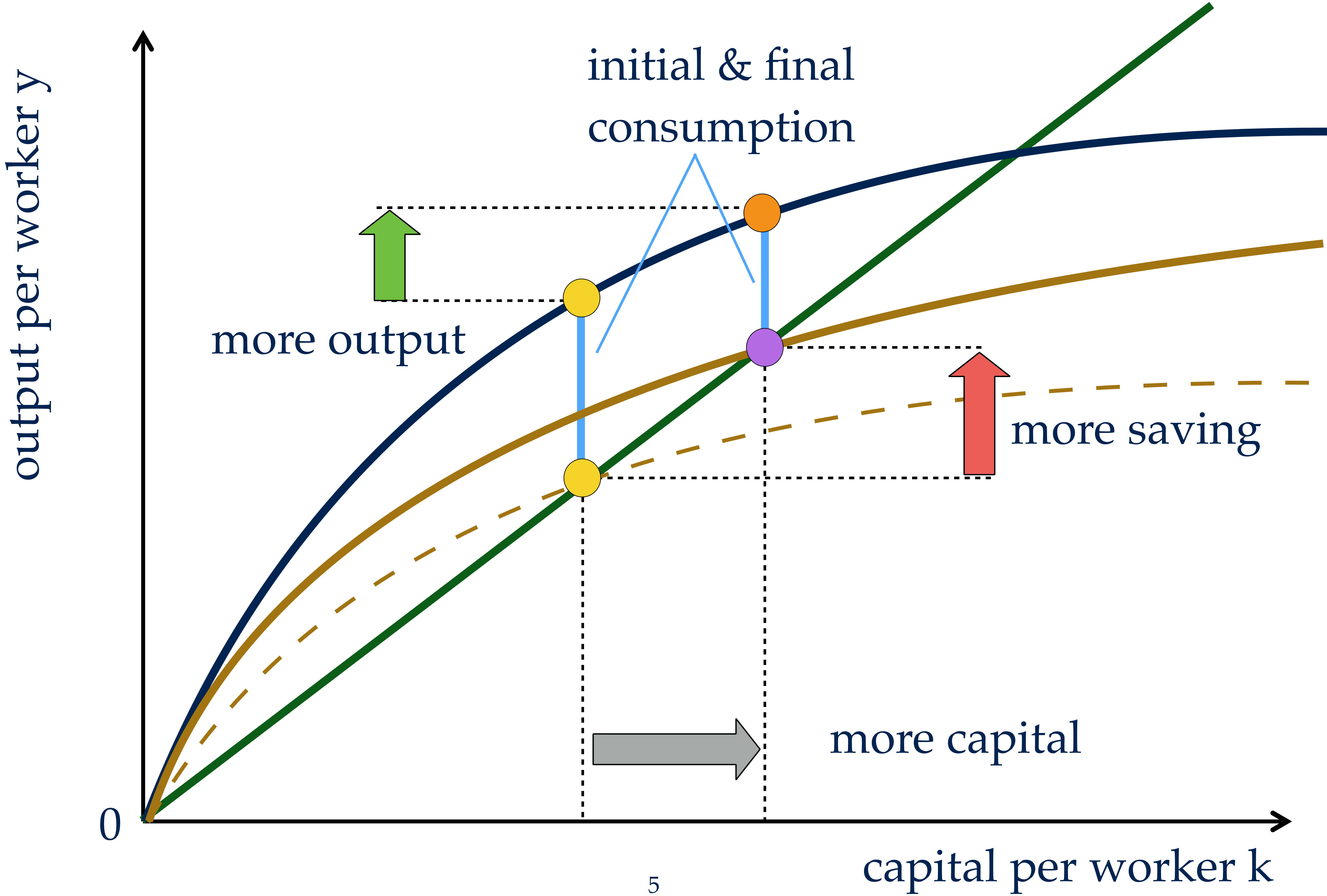
- notation for golden-rule steady state:

  - s_G, $(k^*)_G$, $(c^*)_G$

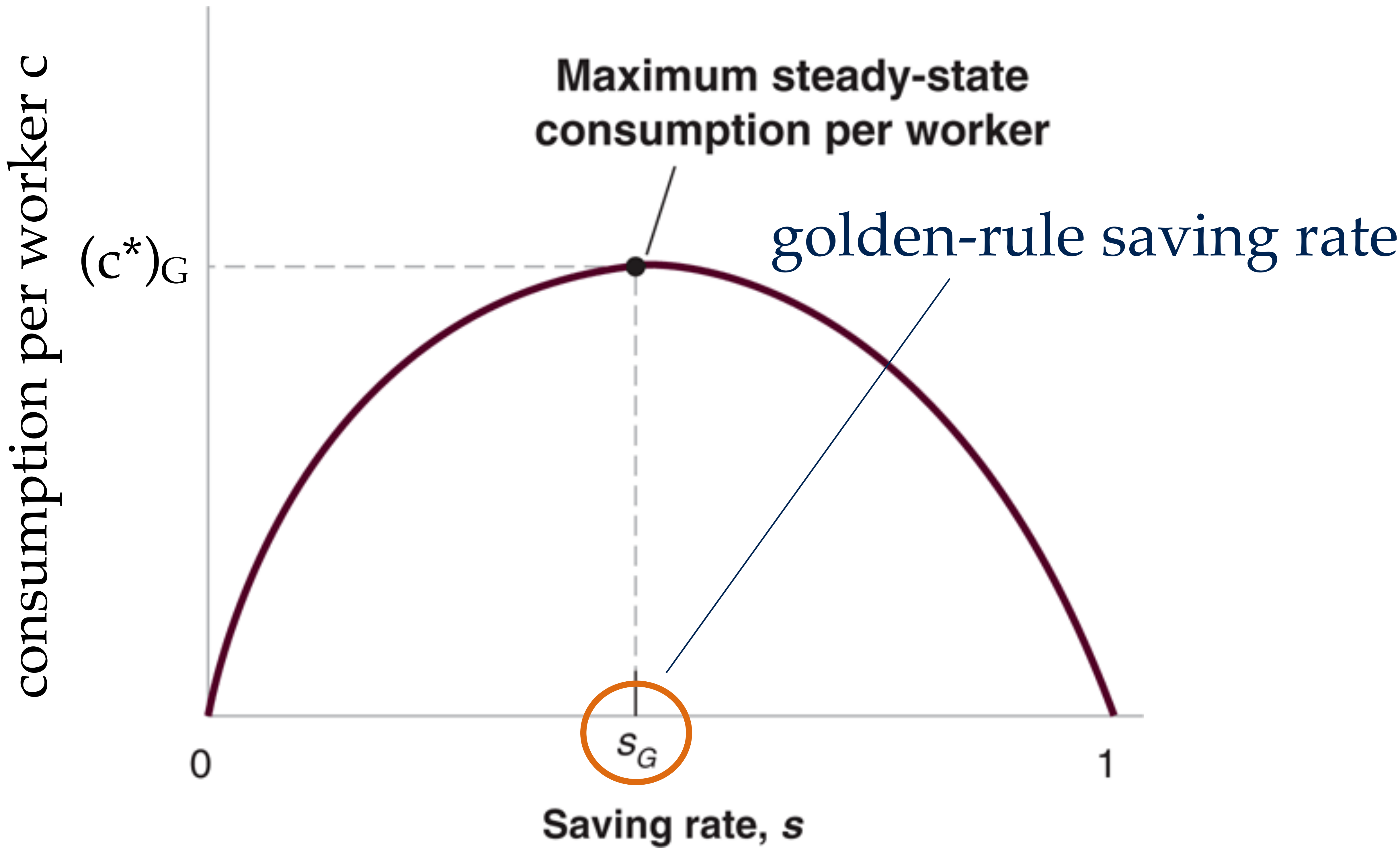
EFFECT OF SAVING RATE ON CONSUMPTION

- what happens when the saving rate s increases?
  - higher s —> higher k* —> higher income $y^* = f(k^*)$ —> higher $c^* = (1 - s) \times y^*$
  - higher s —> lower share of income for consumption (1– s) —> lower $c^* = (1 - s) \times y^*$
- <u>there is a tradeoff:</u>
  - a higher saving rate is good for workers in that it leads to higher output per worker
  - a higher saving rate is bad for workers in that it allocates more output to investment instead of consumption

INCREASE IN SAVING RATE: THE TRADEOFF



GOLDEN RULE: DIAGRAM



IMPLICATIONS OF THE GOLDEN RULE

- for a saving rate below the golden rule, higher saving rate leads to
  - higher capital per worker
  - higher output per worker & investment per worker
  - higher consumption per worker
- for a saving rate above the golden rule, higher saving rate leads to
  - higher capital per worker
  - higher output per worker & investment per worker
  - lower consumption per worker

STEADY STATE: NUMERICAL EXAMPLE

- Cobb-Douglas production function: $F(K,N)=K^{1/2} \times N^{1/2}$ so $f(k) = k^{1/2}$
- law of motion of capital:
  - $k(t+1) - k(t) = s \times k(t)^{1/2} - \delta \times k(t)$
- in steady state: investment = depreciation
  - $s \times (k^*)^{1/2} = \delta \times k^*$ so $k^* = (s/\delta)^2$
  - $y^* = f(k^*)$ so $y^* = s/\delta$
- in the long run, when the saving rate doubles:
  - output per worker doubles
  - capital per worker quadruples

GOLDEN RULE: NUMERICAL EXAMPLE

- steady-state consumption per worker:
  - $c^* = (1 - s) \times y^* = (1 - s) \times s / \delta$
- the golden-rule saving rate maximizes steady-state consumption per worker $c^*(s) = (s - s^2) / \delta$
- the derivative is $dc^*/ds = (1 - 2 \times s) / \delta$
- the golden-rule saving rate satisfies: $dc^*/ds = 0$
- golden rule saving rate: $s_G = 1/2 = 50\%$
- then: $(y^*)_G = 1/(2 \times \delta)$, $(k^*)_G = 1/(4 \times \delta^2)$, $(c^*)_G = 1/(4 \times \delta)$

GOLDEN RULE: ANOTHER CHARACTERIZATION

- $(k^*)_G$ = steady-state capital maximizing steady-state consumption

- link between consumption and capital in steady state:

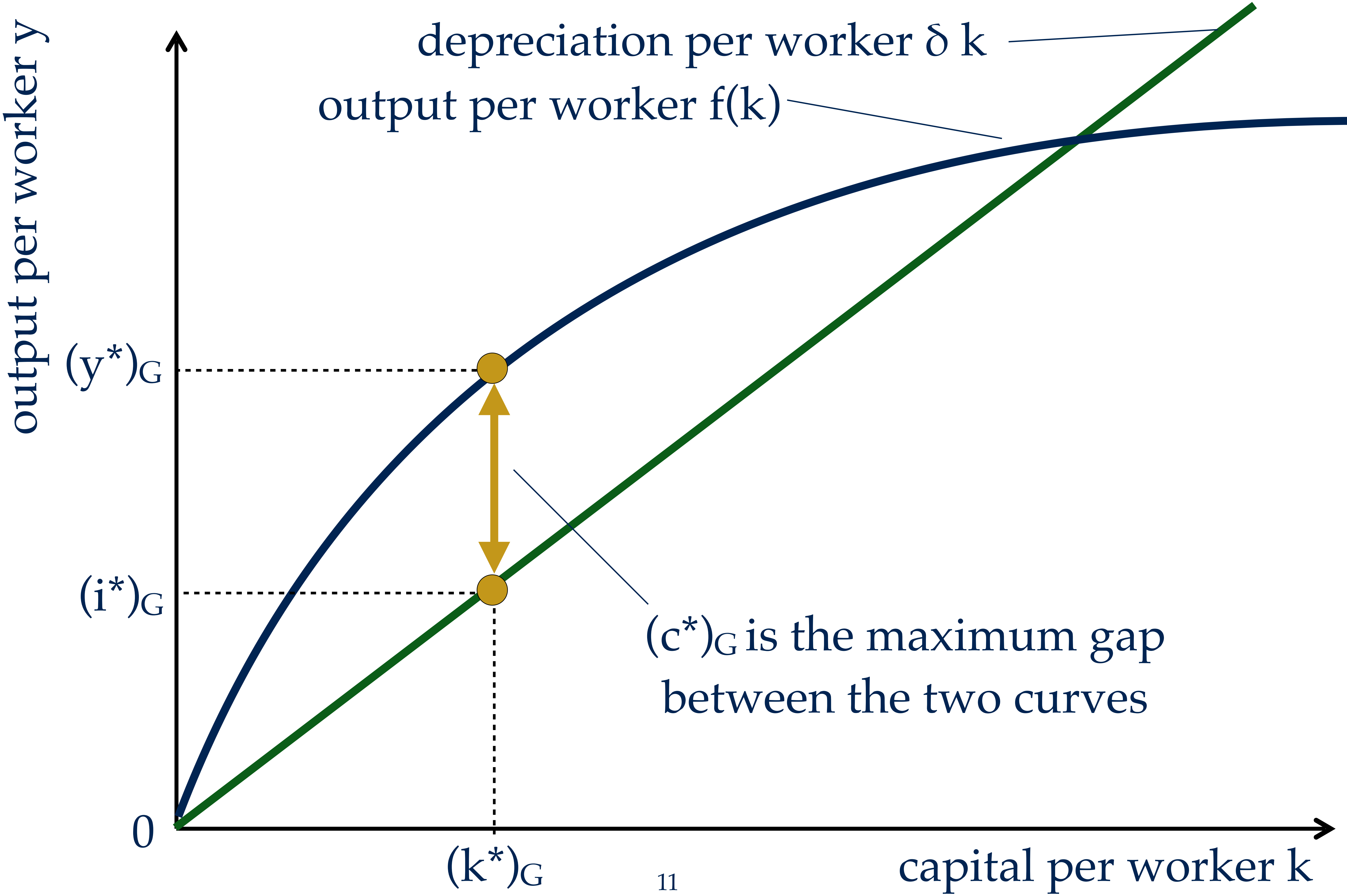
  - output $y^* = f(k^*)$

  - investment $i^* = \text{depreciation} = \delta \times k^*$

  - consumption $c^* = \text{output} - \text{investment} = f(k^*) - \delta \times k^*$

- $(k^*)_G$ maximizes the gap between $f(k^*)$ and $\delta \times k^*$

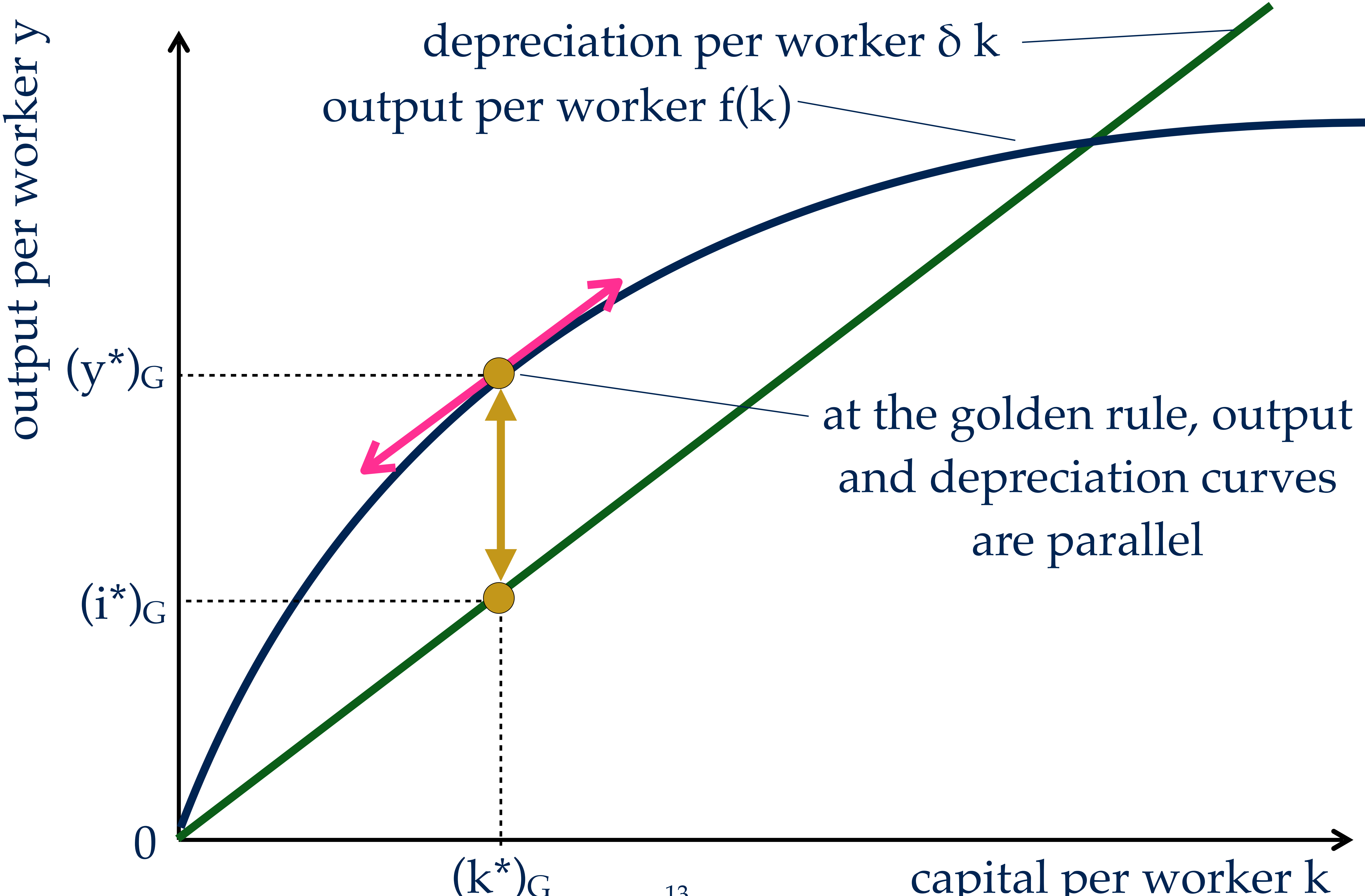
GOLDEN RULE: DIAGRAM



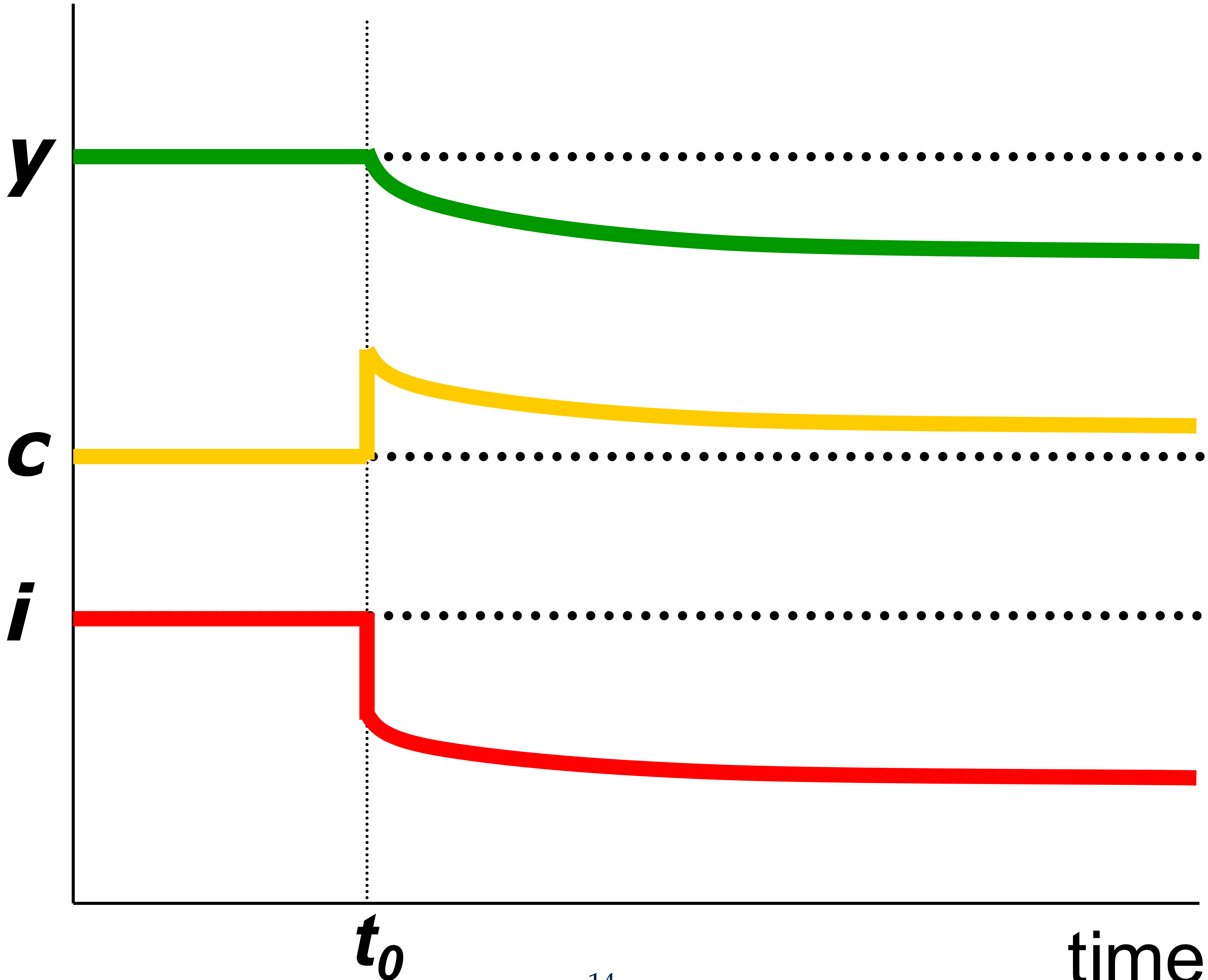
GOLDEN RULE: ANOTHER CHARACTERIZATION

- $(k^*)_G$ = steady-state capital maximizing steady-state consumption
- $(k^*)_G$ maximizes $f(k^*) - \delta \times k^*$
- first-order condition for the maximization:
  - $f'(k^*) = \delta$
- graphical interpretation: the output curve is parallel to the depreciation curve

GOLDEN RULE: DIAGRAM



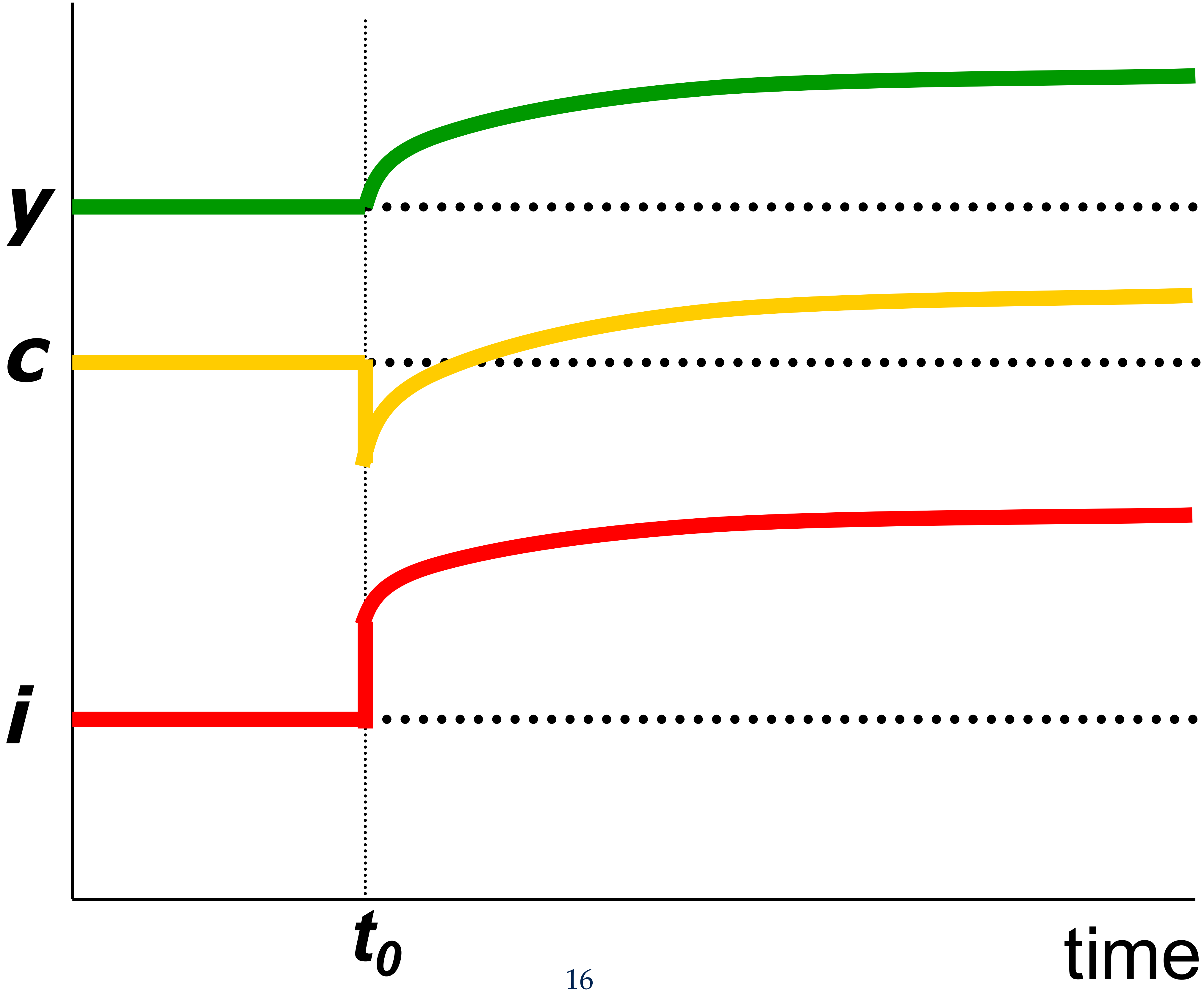
REDUCTION IN SAVING RATE TOWARD GOLDEN RULE



REDUCTION IN SAVING RATE TOWARD GOLDEN RULE

- since s falls toward s_G, in the long run we have:
  - lower y and lower $i = s \times y$
  - but higher c
- y is determined by k, which moves slowly according to its law of motion, so y moves slowly, without jumps
  - in the transition after the reduction in s, y falls slowly as capital depreciates faster than investment
- c and i can jump when s jumps, on the other hand
  - after the reduction in s, since y remains the same initially, $i = s \times y$ jumps down and $c = (1 - s) \times y$ jumps up
  - after the jump, $i = s \times y$ and $c = (1 - s) \times y$ just follow the path of y

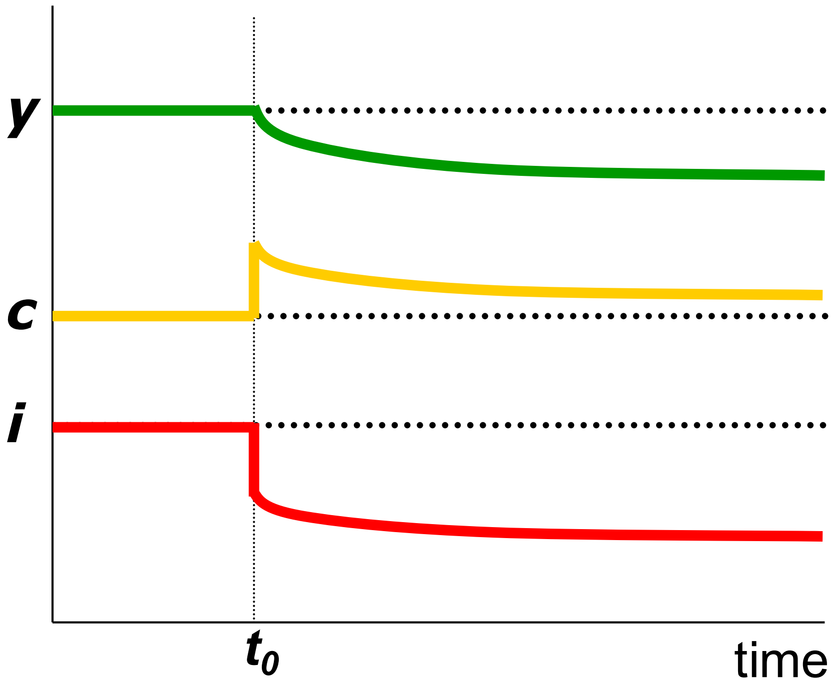
INCREASE IN SAVING RATE TOWARD GOLDEN RULE

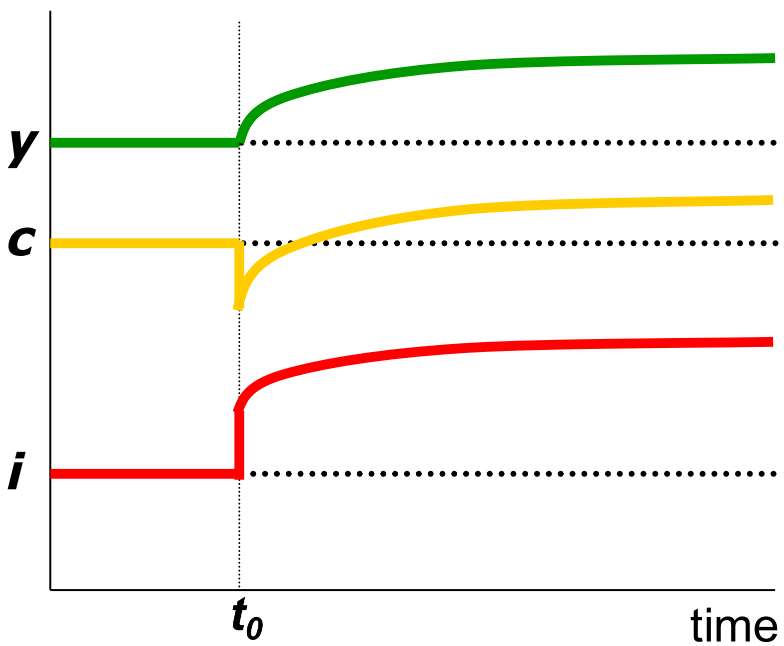


INCREASE IN SAVING RATE TOWARD GOLDEN RULE

- since s rises toward s_G, in the long run we have:
  - higher y and higher $i = s \times y$
  - higher c
- y is determined by k, which moves slowly according to its law of motion, so y moves slowly, without jumps
  - in the transition after the increase in s, y rises slowly as there is more investment than depreciation
- c and i can jump when s jumps, on the other hand
  - after the increase in s, since y remains the same initially, $i = s \times y$ jumps up and $c = (1 - s) \times y$ jumps down
  - after the jump, $i = s \times y$ and $c = (1 - s) \times y$ just follow the path of y

WHICH POLICIES WILL BE IMPLEMENTED?





- decrease in the saving rate from s to s_G: consumption is higher at all points in time
- all generations benefit from the decrease in the saving rate, so policy is easy to implement
- increase in the saving rate from s to s_G : future generations enjoy higher consumption, but the current generation experiences an initial drop in consumption
- not all generations benefit from the increase in the saving rate, so policy is difficult to implement