LECTURE 28
SOLOW | TECHNOLOGICAL PROGRESS

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NOTATION AND ASSUMPTIONS

- we denote by g_X the growth rate of X(t)
  - $g_X = [X(t+1) - X(t)] \ / \ X(t)$
- assumption 1: technology A grows at rate g_A
- assumption 2: working population N grows at rate g_N
- once we have technological growth, having population growth makes the model richer without complicating it

PRODUCTION FUNCTION WITH TECHNOLOGICAL PROGRESS

- we focus on the production function: $Y = F(K, A \times N)$
  - Y: output
  - K: capital
  - N: labor
  - A: labor-augmenting technology
  - $A \times N$: effective labor
  - 2 factors of production: capital and effective labor

PROPERTIES OF PRODUCTION FUNCTION

- increasing in K and $A \times N$
- constant returns to scale
  - $F(b \times K, b \times AN) = b \times F(K,AN)$ for any scalar b
- decreasing returns to capital

OUTPUT AND CAPITAL PER EFFECTIVE WORKER

- by constant returns to scale, there is a relation between
  - output per effective worker $y = Y/AN$
  - capital per effective worker $k = K/AN$
- $y=F(K,AN)/AN = F(K/AN,AN/AN) = F(k,1) = f(k)$
  - the function f is such that $f(x) = F(x,1)$
- the function $f(k)$ is increasing and concave ($f'(k)>0$ and $f''(k)<0$)
  - higher capital per effective worker leads to higher output per effective worker, but at a decreasing rate

SAVING AND INVESTMENT

- assumption 1: the economy is closed so investment = private saving + public saving
- assumption 2: no public saving so private saving = investment
- assumption 3: private saving depends on income: $S = s \times Y$
  - s is the saving rate
- hence, investment depends on output: $I = s \times Y$
- thus, investment per effective worker ($i = I/AN$) depends on capital per effective worker:
  - $i = I/AN = s \times Y/AN = s \times y = s \times f(k)$

GROWTH RATE OF CAPITAL

- evolution of the capital stock is driven by investment and depreciation: capital tomorrow = capital today + investment today – depreciation today

- law of motion of capital:
  - $K(t+1) = K(t) + I(t) - \delta \times K(t)$
  - $K(t+1) - K(t) = I(t) - \delta \times K(t)$

- growth rate of capital:
  - $g_K = [K(t+1) - K(t)] \;/\; K(t) = [I(t) \;/\; K(t)] - \delta$

GROWTH RATE OF CAPITAL PER EFFECTIVE WORKER

- recall: the growth rate of the product of two variables is the sum of the growth rates of each variable

- capital per effective worker is $k = K / AN$

- so $g_k = g_K - g_A - g_N$

LAW OF MOTION OF CAPITAL PER EFFECTIVE WORKER

- using previous results:
  - $k(t+1) - k(t) = g_k \times k(t)$ (definition of growth rate)
  - $k(t+1) - k(t) = [g_K - g_A - g_N] \times k(t)$ (result on g_k)
  - $k(t+1) - k(t) = g_K \times k(t) - [g_A + g_N] \times k(t)$
- we saw that $g_K = \{I(t) / K(t)\} - \delta$
  - hence $g_K = \{I(t) / [k(t) \times A(t)N(t)]\} - \delta$
- so $g_K \times k(t) = [\ I(t) / A(t)N(t)\ ] - \delta \times k(t) = i(t) - \delta \times k(t)$

LAW OF MOTION OF CAPITAL PER EFFECTIVE WORKER

- we have:
  - $k(t+1) - k(t) = g_K \times k(t) - [g_A + g_N] \times k(t)$
  - $g_K \times k(t) = i(t) - \delta \times k(t)$
- so we infer: $k(t+1) - k(t) = i(t) - [\delta + g_A + g_N]\, k(t)$
- next, investment per effective worker = saving per effective worker:
  - $i(t) = s \times f(k(t))$
- to conclude: $k(t+1) - k(t) = s \times f(k(t)) - [\delta + g_A + g_N] \times k(t)$
- same as in basic Solow model, but δ is replaced by $\delta + g_A + g_N$