LECTURE 29
SOLOW | BALANCED GROWTH

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LAW OF MOTION OF CAPITAL PER EFFECTIVE WORKER

- using the definition of growth rate g_k:
  - $k(t+1) - k(t) = g_k \times k(t)$
- first step: compute g_k
  - since $k = K \;/\; AN$
  - then $g_k = g_K - (g_A + g_N)$
- hence $k(t+1) - k(t) \;= [g_K - (g_A + g_N)] \times k(t)$
- which implies: $k(t+1) - k(t) \;= g_K \times k(t) - [g_A + g_N] \times k(t)$

GROWTH RATE OF CAPITAL

- evolution of the capital stock is driven by investment and depreciation: capital tomorrow = capital today + investment today – depreciation today
  - $K(t+1) - K(t) = I(t) - \delta \times K(t)$
- growth rate of capital:
  - $g_K = [K(t+1) - K(t)] \ / \ K(t) = [I(t) \ / \ K(t)] - \delta$
- since $K(t) = k(t) \times A(t)N(t)$, we conclude that
  - $g_K \times k(t) = I(t) / [A(t)N(t)] - \delta \times k(t) = i(t) - \delta \times k(t)$

BACK TO LAW OF MOTION OF CAPITAL PER EFFECTIVE WORKER

- we have:

  1. $k(t+1) - k(t) = g_K \times k(t) - [g_A + g_N] \times k(t)$

  2. $g_K \times k(t) = i(t) - \delta \times k(t)$

  3. $i(t) = s \times f(k(t))$

- hence: $k(t+1) - k(t) = s \times f(k(t)) - [\delta + g_A + g_N] \times k(t)$

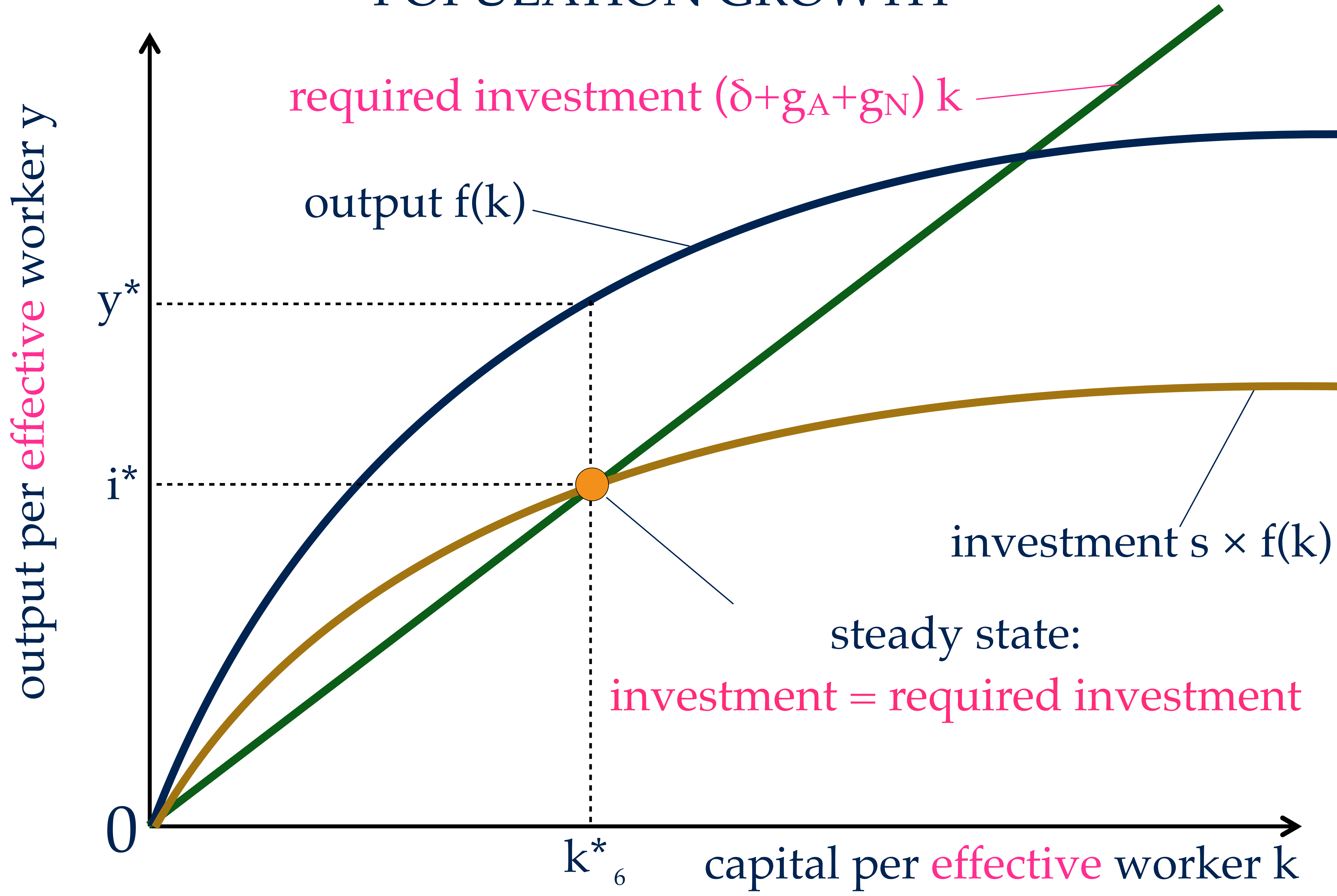
- same law of motion as in basic Solow model

  - but δ is replaced by $\delta + g_A + g_N$

THE STEADY STATE

- capital per effective worker is constant
- output per effective worker is constant
- using the law of motion of capital per effective worker, we find that steady-state capital per effective worker k^* satisfies
  - $s \times f(k^*) = [\delta + g_A + g_N] \times k^*$
- to maintain $k = K/AN$ constant, there must be enough investment
  - to cover depreciation of K (δ)
  - to cover growth of A (g_A) and growth of N (g_N)

EQUILIBRIUM WITH TECHNOLOGICAL & POPULATION GROWTH



BALANCED GROWTH IN STEADY STATE

definition of steady state

k^*

y^*

	Growth Rate:
Capital per effective worker	0
Output per effective worker	0
Capital per worker	g_A
Output per worker	g_A
Labor	g_N
Capital	$g_A + g_N$
Output	$g_A + g_N$

BALANCED GROWTH IN STEADY STATE

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growth rate of population is given

BALANCED GROWTH IN STEADY STATE

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Output per effective worker	0
Capital per worker	g_A
Output per worker	g_A
Labor	g_N
Capital	$g_A + g_N$
Output	$g_A + g_N$

$(K/N)^* = k^* \times A$

$(Y/N)^* = y^* \times A$

there is balanced growth because several variables grow at the same rate

BALANCED GROWTH IN STEADY STATE

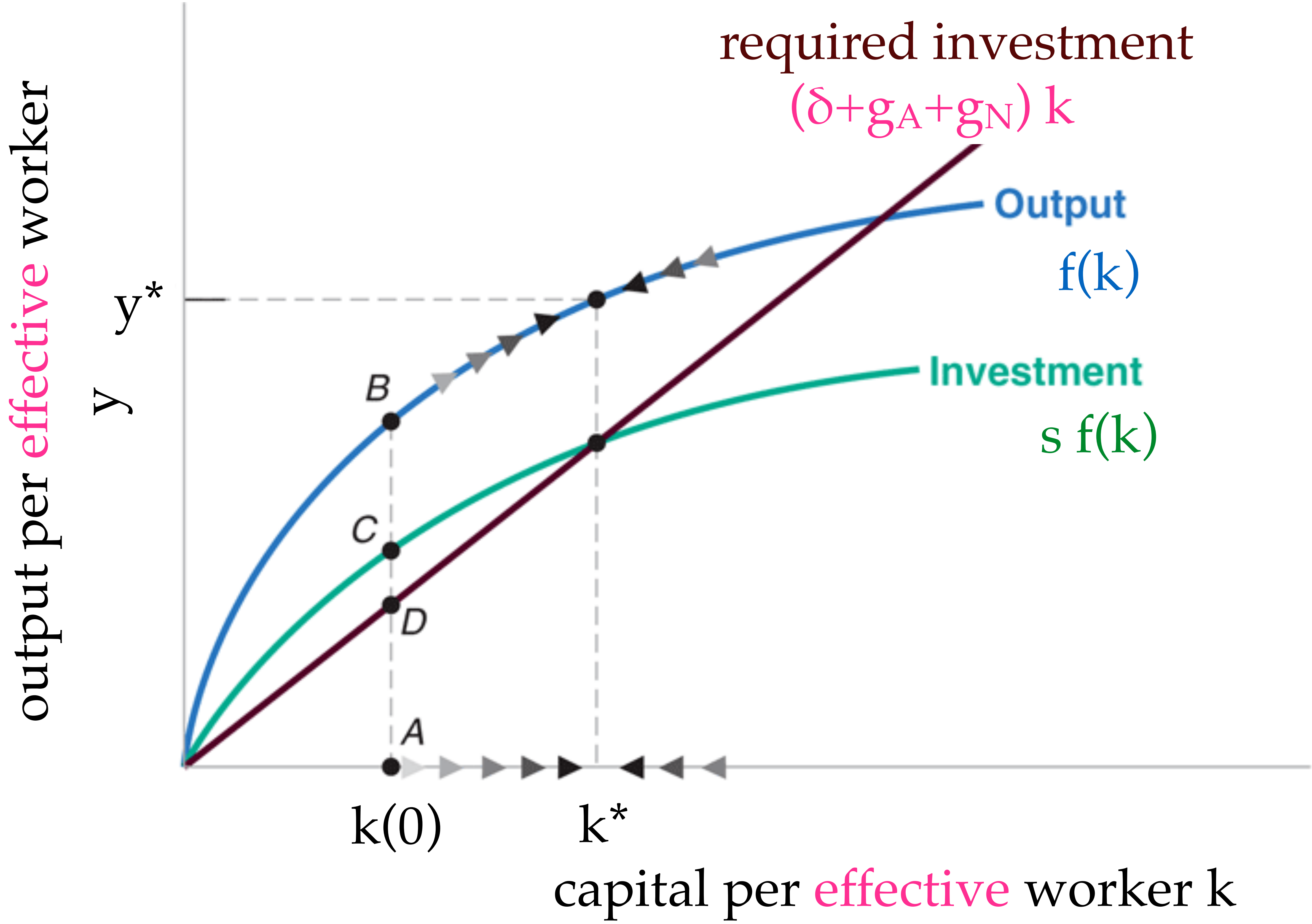
	Growth Rate:
Capital per effective worker	0
Output per effective worker	0
Capital per worker	g_A
Output per worker	g_A
Labor	g_N
Capital	$g_A + g_N$
Output	$g_A + g_N$

$$K^* = k^* \times A \times N$$

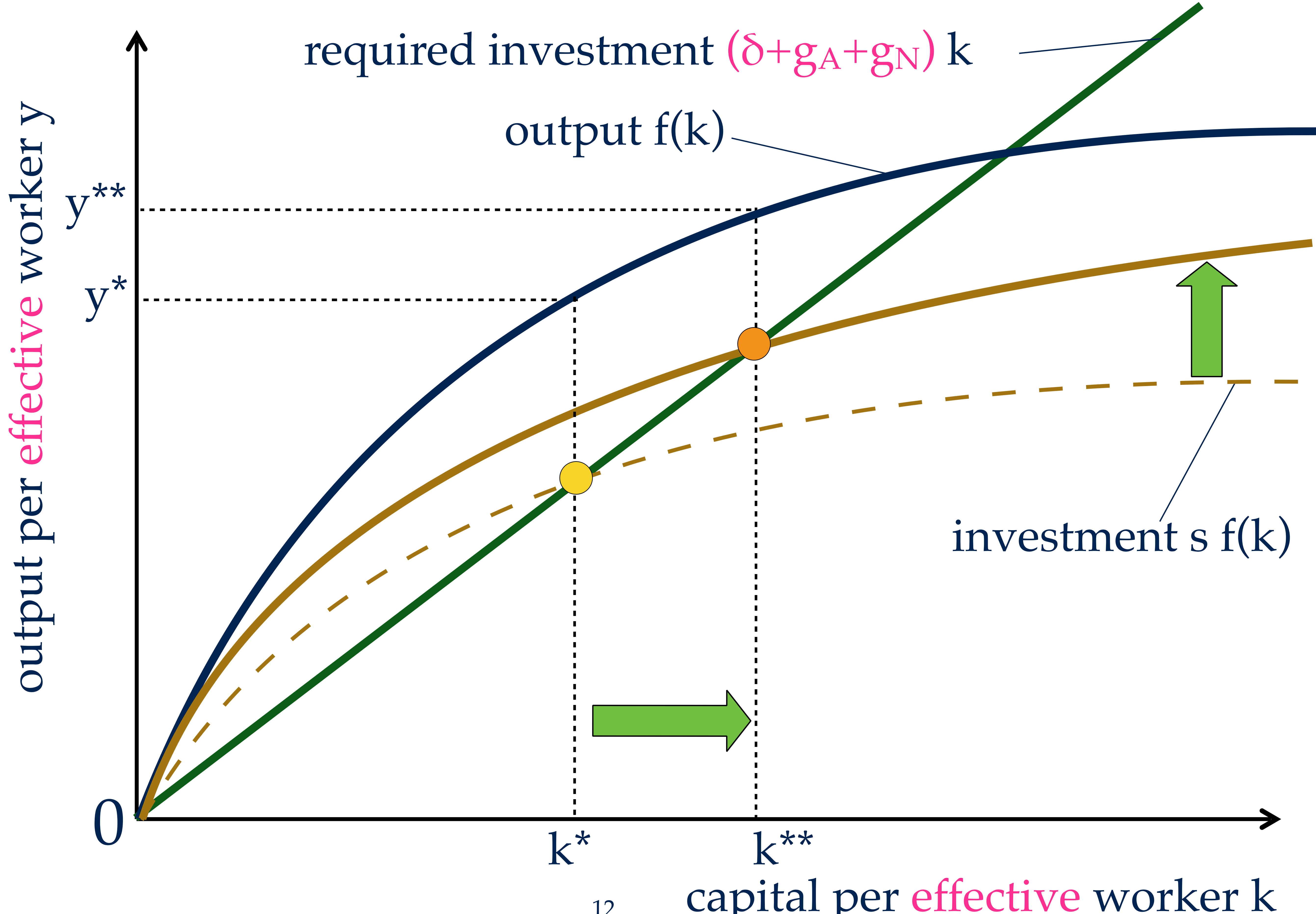
$$Y^* = y^* \times A \times N$$

there is balanced growth because several variables grow at the same rate

EQUILIBRIUM DIAGRAM



INCREASE IN SAVING RATE: STEADY STATE



INCREASE IN THE SAVING RATE: DYNAMICS