LECTURE 16
MATCHING | LABOR DEMAND

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LABOR DEMAND: DEFINITION

- labor demand measures the number of workers that firms want to employ for a given wage and tightness

- labor demand depends on how productive workers are, how costly it is to employ workers (wage), and how easy it is to recruit new workers (tightness)

TWO TYPES OF WORKER

- keeping a vacancy open for a month requires r recruiters
  - $r > 0$ is a parameter
- hence there are two types of worker in firms:
  - N producers: produce goods and services
  - R recruiters: fill vacancies by creating job descriptions, advertising vacancies, selecting applicants, reading CVs, conducting interviews
- total number of workers: $L = N + R$

RECRUITER-PRODUCER RATIO

- the recruiter-producer ratio is $\tau(\theta) = R/N$
- number of producers and total number of workers are related by the recruiter-producer ratio
  - $L = N + R = N + N \times \tau(\theta)$
  - so $L = (1 + \tau(\theta)) \times N$
- when the recruiter-producer ratio is high, there is a larger gap between total number of workers and number of producers

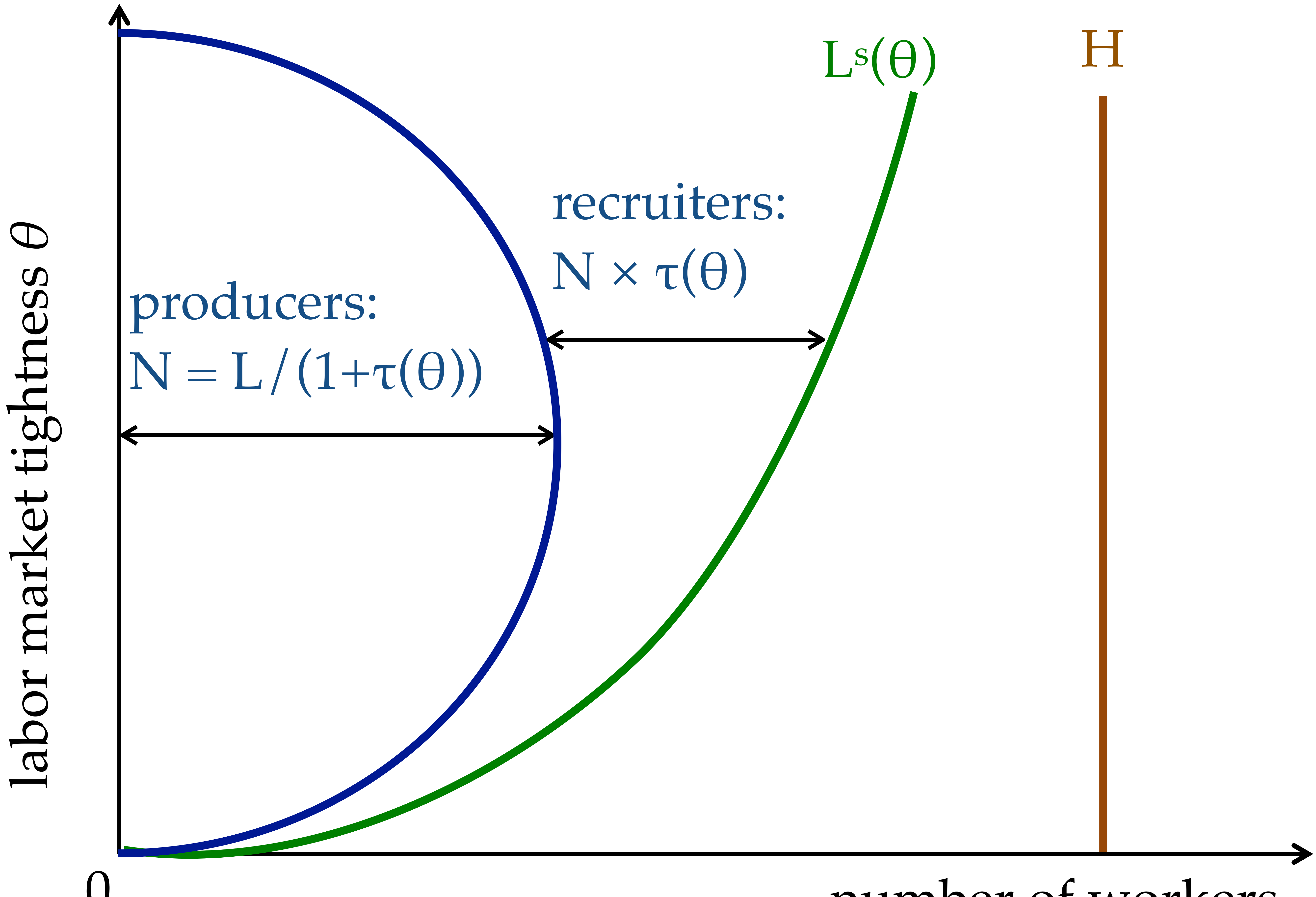
NUMBER OF RECRUITERS IN LABOR MARKET

- $s \times L$ jobs are destroyed each month, so with balanced flows $s \times L$ jobs need to be created
- vacancies are filled with probability $q(\theta)$, so if V vacancies are posted, $q(\theta) \times V$ jobs are created
- to fill $s \times L$ jobs, it is therefore necessary to post a number $V = L \times s \,/\, q(\theta)$ of vacancies
- V vacancies require $r \times V$ recruiters, so the number of recruiters is $R = r \times s \times L / q(\theta)$

LINK BETWEEN RECRUITER-PRODUCER RATIO AND TIGHTNESS

- Given that $R = r \times L \times s \;/\; q(\theta)$, we have:
  - $R \times q(\theta) = (R+N) \times r \times s$
  - $(R/N) \times q(\theta) = (R/N+1) \times r \times s$
  - $\tau(\theta) \times [q(\theta) - r \times s] = r \times s$
  - so $\tau(\theta) = (r \times s)/[q(\theta) - (r \times s)]$
- (assumption: θ is low enough so $q(\theta) - r \times s > 0$ and $\tau(\theta) > 0$)
- property: the recruiter-producer ratio $\tau(\theta)$ is increasing in θ
  - because $q(\theta)$ is decreasing in θ
  - when tightness is higher, it is more difficult to fill vacancies, so firms have to allocate more workers to recruiting

SUPPLY OF RECRUITERS AND PRODUCERS



FIRMS

- output of a firm is given by its production function = $a \times N^{\alpha}$
  - α is between 0 and 1
  - a represents the productivity of the firm
  - N: number of producers employed by the firm
- firms pay a wage W to its L workers (recruiters + producers)
  - total labor costs: $L \times W = [1+ \tau(\theta)] \times N \times W$
  - labor cost per producer: $(1+ \tau(\theta)) \times W$
- firm's profits = production minus labor costs
  - profits $= a \times N^{\alpha} - [1+\tau(\theta)] \times W \times N$

LABOR DEMAND: DERIVATION

- to maximize profits, the derivative of profits with respect to N must be 0
  - profits: $a \times N^{\alpha} - [1+\tau(\theta)] \times W \times N$
  - derivative: $\alpha \times a \times N^{\alpha-1} - W \times (1+\tau(\theta)) = 0$
- this implies $N^{\alpha-1} = W \times [1+\tau(\theta)] / [\alpha \times a]$
- therefore the optimal number of producers for the firm is

$$N = \left[\frac{\alpha \cdot a}{W \cdot (1 + \tau(\theta))}\right]^{1/(1-\alpha)}$$

LAST STEP TO OBTAIN LABOR DEMAND

$$N = \left[\frac{a \cdot \alpha}{W \cdot [1 + \tau(\theta)]} \right]^{1/(1-\alpha)}$$

$$L = [1 + \tau(\theta)] \cdot \left[\frac{a \cdot \alpha}{W \cdot [1 + \tau(\theta)]} \right]^{1/(1-\alpha)}$$

$$L = \cdot \left[\frac{a \cdot \alpha \cdot [1 + \tau(\theta)]^{1-\alpha}}{W \cdot [1 + \tau(\theta)]} \right]^{1/(1-\alpha)}$$

$$L = \cdot \left[\frac{a \cdot \alpha}{W \cdot [1 + \tau(\theta)]^{\alpha}} \right]^{1/(1-\alpha)}$$

LABOR DEMAND: EXPRESSION

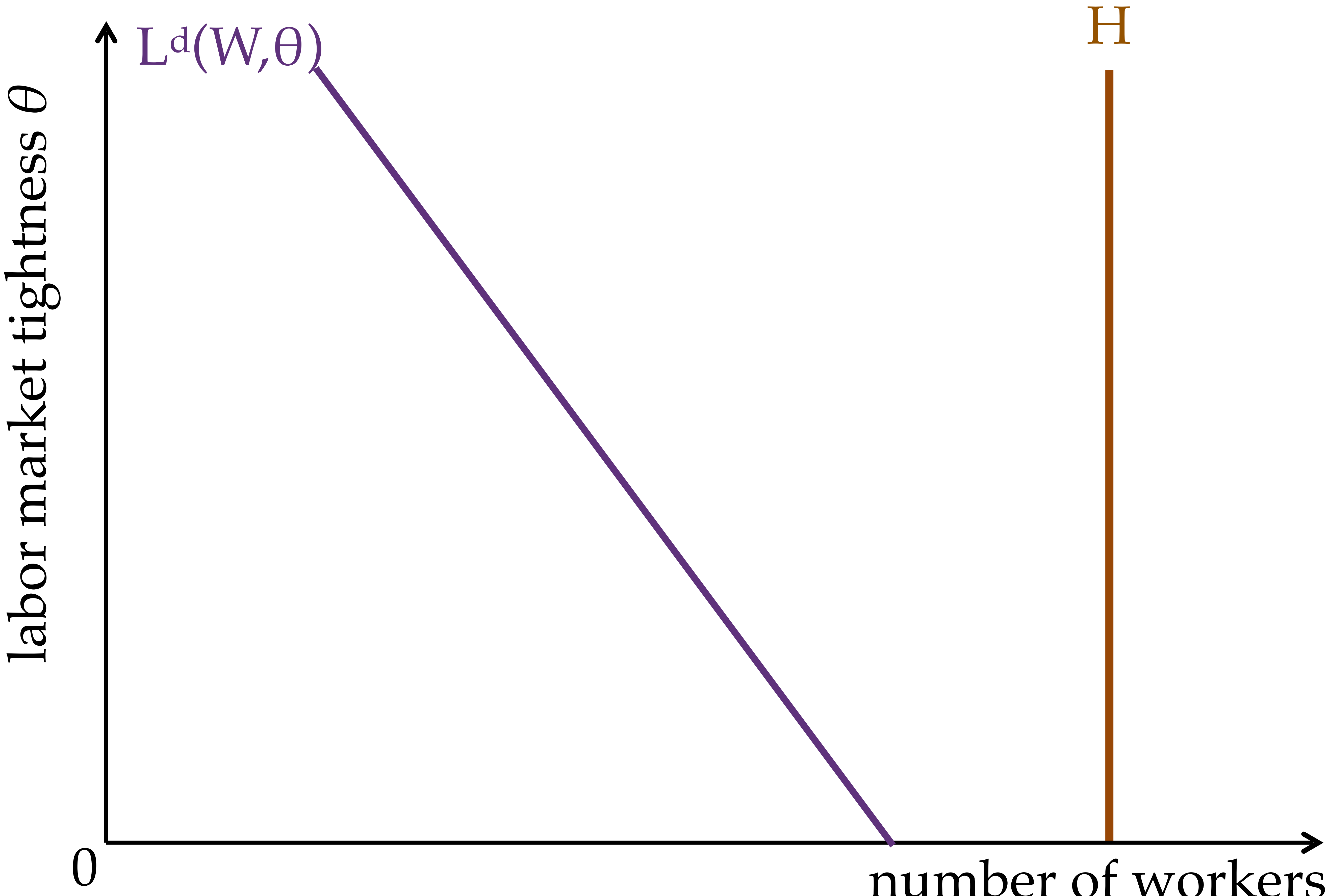
$$L^d(\theta, W) = \left[\frac{\alpha \cdot a}{W \cdot (1 + \tau(\theta))^{\alpha}}\right]^{1/(1-\alpha)}$$

- it is obtained by multiplying the optimal number of producers N by $(1+\tau(\theta))$
- this is the expression of the labor demand: the optimal number of workers (recruiters + producers) that firms want to hire

LABOR DEMAND: PROPERTIES

- the profitability of employing workers depends negatively on
  - the wage paid to workers (W)
  - the cost or recruiting workers, which is governed by recruiter-producer ratio ($\tau(\theta)$)
- hence the labor demand $L^d(W,\theta)$
  - is decreasing in W
  - is decreasing in θ (as $\tau(\theta)$ is increasing in θ)

LABOR DEMAND



INCREASE IN WAGES