

LECTURE 22

MALTHUS | OVERVIEW

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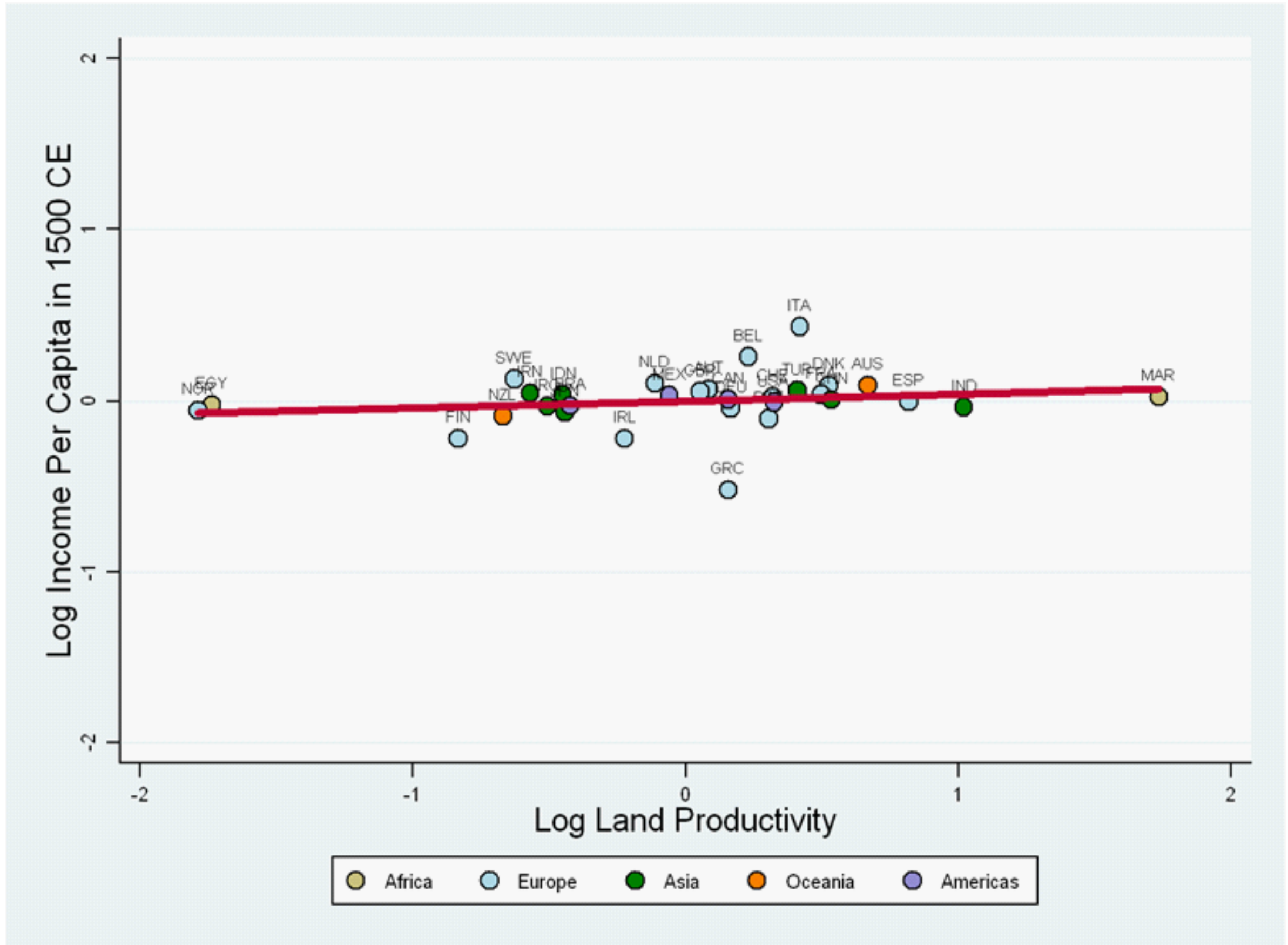
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US GROWTH AFTER INDUSTRIAL REVOLUTION

- between 1870 and today, US growth = 2% per year
 - especially fast since 1950
- US real GDP: $\text{GDP}(2014) = 46 \times \text{GDP}(1890)$
 - much of it driven by population growth
- US real GDP / person: $\text{GDP}(2014) = 9 \times \text{GDP}(1890)$
 - due to better technology: inventions, production and management techniques, infrastructure, legal and political institutions

...BUT HAS NO EFFECT ON INCOME PER CAPITA!



THE MALTHUSIAN MODEL

- objective: understand how technology advancements may lead to higher population but same living standards
- overlapping generations (two generations live simultaneously)
 - people are born, live two periods, and die
 - at the beginning of each period, new people are born
 - at the end of each period, existing people die
- people produce and consume one good: food
- two factors of production: labor + land

FACTORS OF PRODUCTION

- land X : fixed over time
- technology A : fixed over time
 - percentage of arable land, soil quality, climate, cultivation and irrigation methods, crops available
- labor L : evolves over time
 - determined by the rate of population growth
 - in turn determined by workers' fertility

COBB-DOUGLAS PRODUCTION FUNCTION

- $Y(t) = (A X)^\alpha \times L(t)^{1-\alpha}$
- A: technology
- X: amount of land
- L(t): number of workers in period t
- Y(t): output in period t
- $0 < \alpha < 1$: parameter of the production function
- more workers, more land, better technology yield more output

OUTPUT PER WORKER

- $y(t)$: output produced by a worker in period t
 - $y(t) = Y(t) / L(t) = [A X / L(t)]^\alpha$
- more land and better technology lead to a higher output per worker
- but more workers lead to lower output per worker
 - this is because the amount of land is fixed, so with more workers, each worker farms a smaller plot and therefore produces less output

THE LIFE OF WORKERS

- workers live 2 periods in households organized around one parent
- childhood period: the worker is a child and does not work
 - a child consumes an amount p of food that she receives from her one parent
- adulthood period: the worker is a parent and works
 - a parent decides how many children to have
 - a parent then produces food, consumes some of it, and give some of it to her children

COBB-DOUGLAS UTILITY FUNCTION

- $u(t) = n(t)^\beta \times c(t)^{1-\beta}$
 - $u(t)$: utility enjoyed by a worker
 - $n(t)$: number of children in the worker's household
 - $c(t)$: worker's consumption
 - $0 < \beta < 1$: parameter of the utility function describing how much workers value having children
- a worker enjoys consuming more food and having more children

WORKER'S BUDGET CONSTRAINT

- the budget constraint says that the worker splits her food production between herself and her children
- $y(t) = p \times n(t) + c(t)$
 - $y(t)$: food produced by a worker
 - $c(t)$: food consumed by a worker
 - $p \times n(t)$: food consumed by the worker's $n(t)$ children
- for a worker, having more children brings utility but reduces personal consumption

NUMBER OF CHILDREN THAT WORKERS CAN AFFORD

- a worker must split her food production between herself and her children: $y(t) = p \times n(t) + c(t)$
- hence, for a production $y(t)$ and a consumption $c(t)$, a worker can only afford:
 - $n(t) = [y(t) - c(t)] / p$ children
- a worker can support fewer children when
 - the worker or the children consume more
 - output per worker is lower