# LECTURE 24
# MALTHUS | OUTPUT PER WORKER

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# EVOLUTION OF OUTPUT PER WORKER

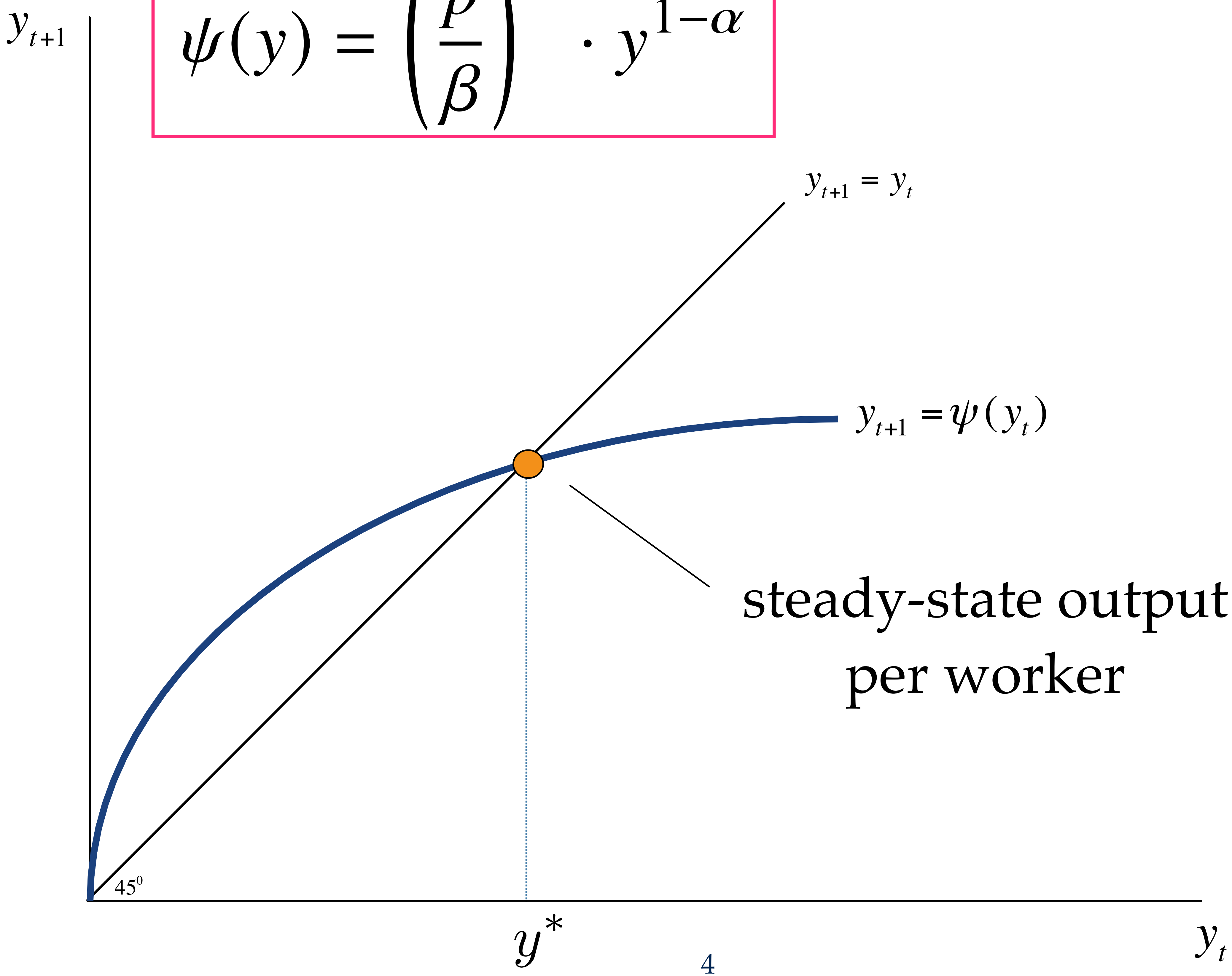
- recall 1: output per worker is $y(t) = [A\ X / L(t)]^{\alpha}$
- recall 2: the fertility rate is $n(t) = (\beta / p) \times y(t)$
- recall 3: population follows $L(t+1) = n(t) \times L(t)$
- hence: $y(t+1) = [A\ X / L(t+1)]^{\alpha}$
- $y(t+1) = [A\, X / (n(t) \times L(t))]^{\alpha} = [A\, X / L(t)]^{\alpha} \times n(t)^{-\alpha}$
- so $y(t+1) = y(t) \times n(t)^{-\alpha}$
- thus $y(t+1) = y(t) \times (\beta / p)^{-\alpha} \times y(t)^{-\alpha} = (p / \beta)^{\alpha} \times y(t)^{1-\alpha}$
- law of motion of output per worker: $y(t+1) = (p / \beta)^{\alpha} \times y(t)^{1-\alpha}$

# STEADY-STATE OUTPUT PER WORKER

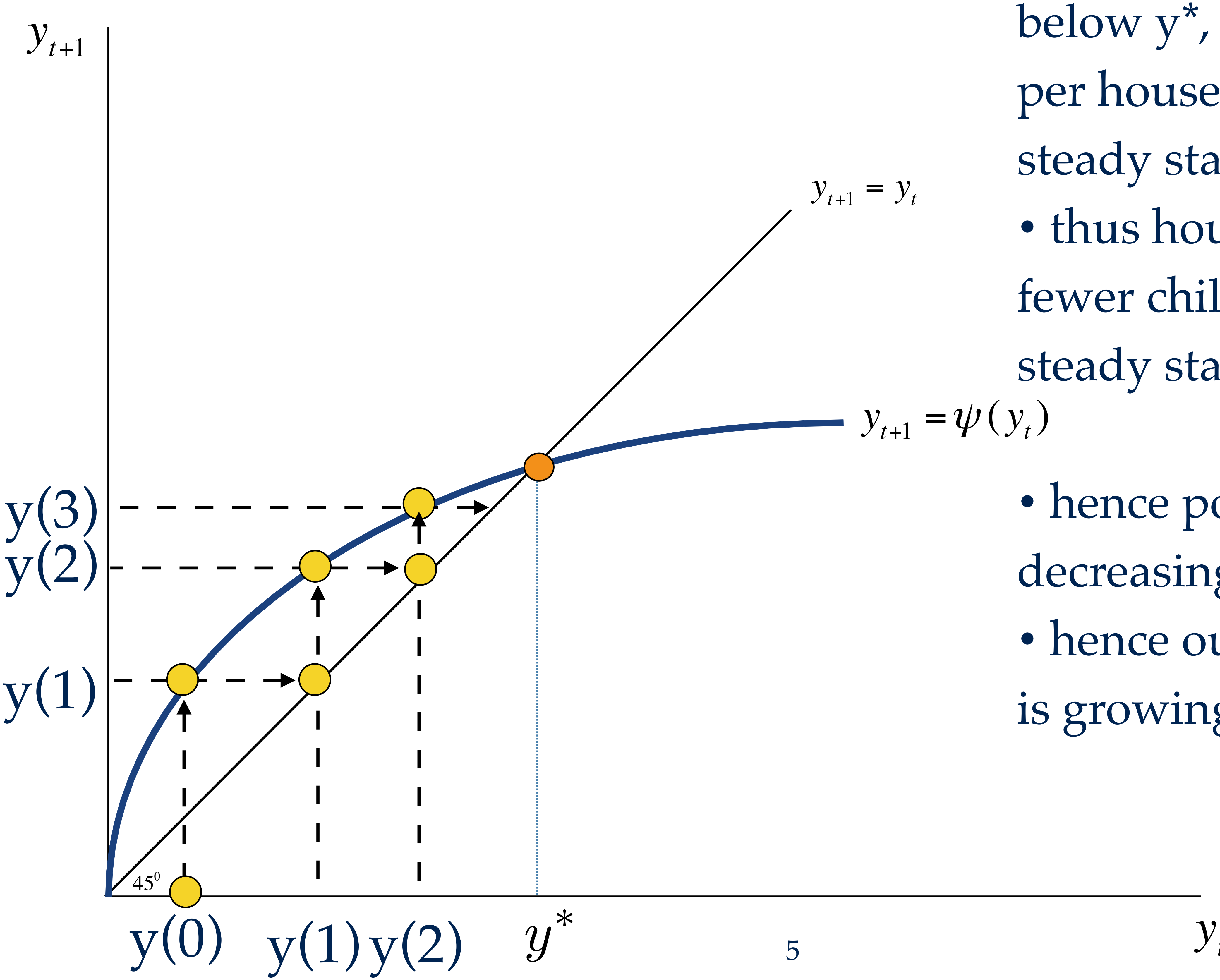
- the dynamics of output per worker are given by $y(t+1) = \psi(y(t))$
  - where the function $\psi(y) = (p/\beta)^{\alpha} \times y^{1-\alpha}$
- steady-state output per worker satisfies:
  - $y(t+1) = y(t)$
  - once output per worker reaches its steady-state level, it does not change
- so in steady state, $y^* = \psi(y^*)$
- $y^* = (p/\beta)^{\alpha} \times (y^*)^{1-\alpha}$
- hence in steady state: $y^* = p/\beta$

# STEADY-STATE OUTPUT PER WORKER

$$\psi(y) = \left(\frac{p}{\beta}\right)^{\alpha} \cdot y^{1-\alpha}$$



# DYNAMICS OF OUTPUT PER WORKER



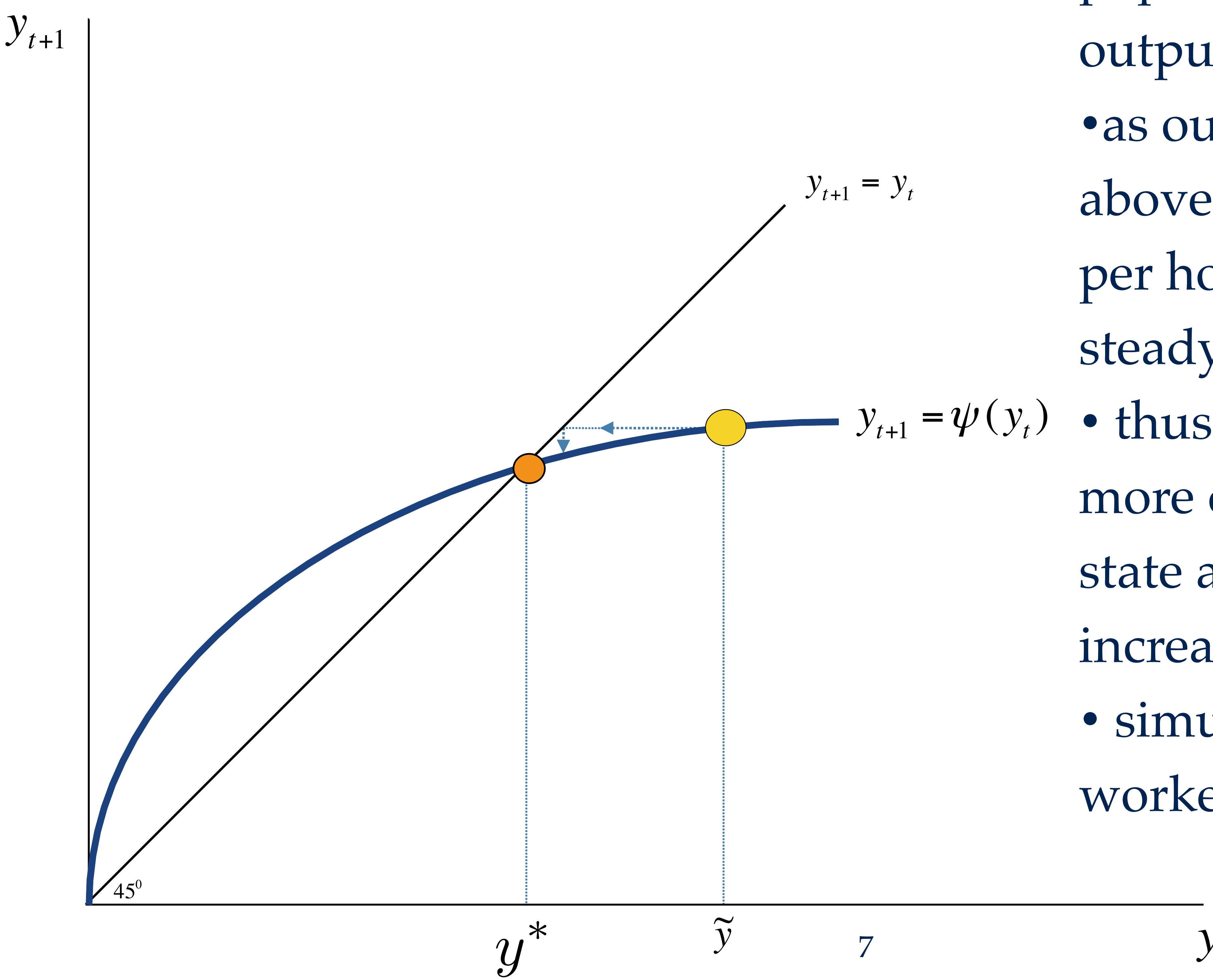

- as output per worker is below y*, there is less food per household than in steady state
- thus households have fewer children than in steady state

- hence population is decreasing over time
- hence output per worker is growing over time

# DETERMINANTS OF LONG-RUN OUTPUT PER WORKER

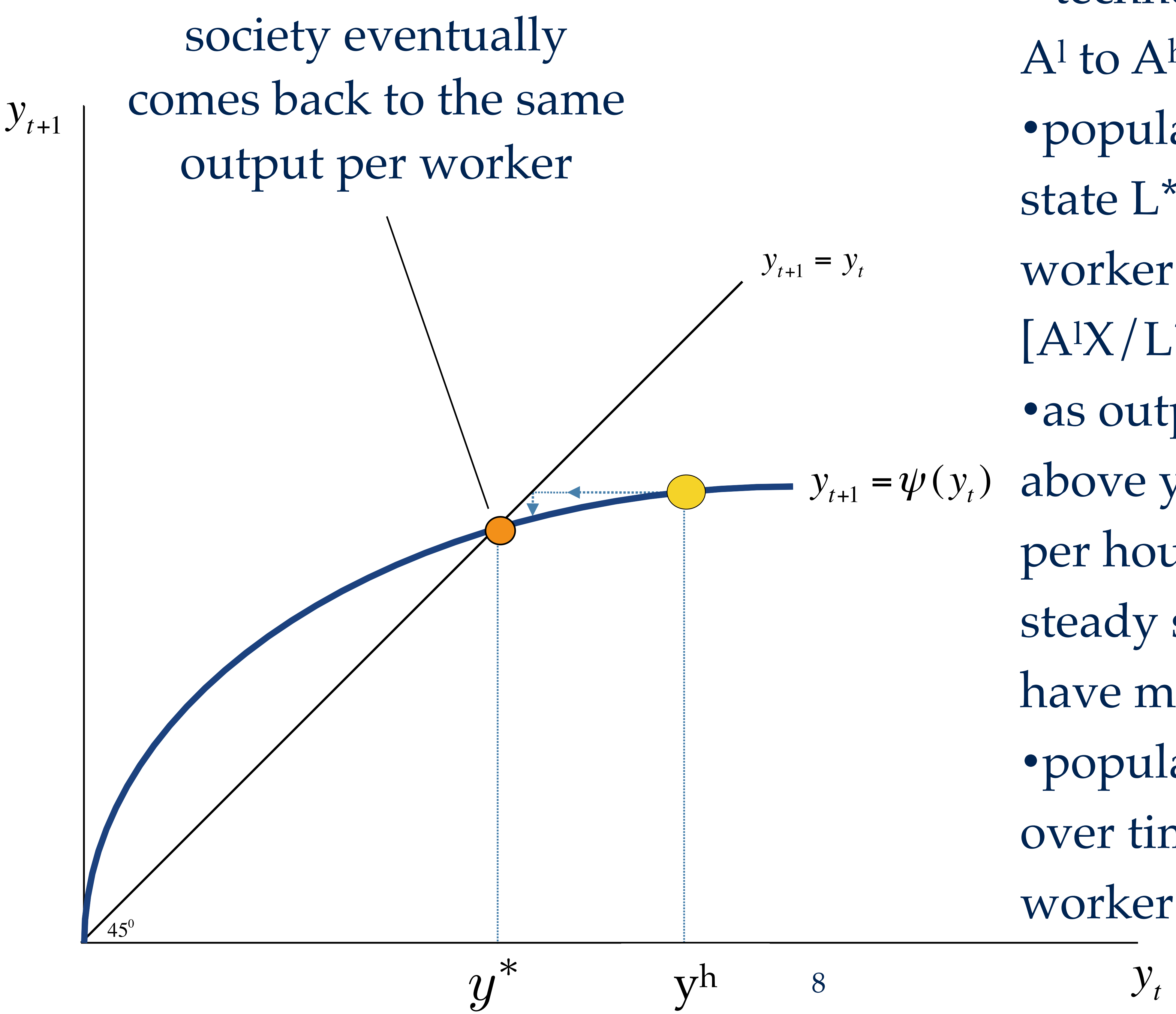
- output per worker y determines the standards of living
  - because each worker consumes $c = (1-\beta) \times y$
  - (each child consumes a fixed amount p)
- steady-state output per worker is higher when
  - people value children less (low $\beta$)
  - children eat more food (high p)
  - but land (X) and technology (A) have no effect on output per worker in steady state

# DYNAMICS AFTER AN EPIDEMICS




- after an epidemics, the population shrinks, so output per worker increases
- as output per worker is above y*, there is more food per household than in steady state
- thus households have more children than in steady state and population is increasing over time
- simultaneously output per worker is falling over time

# IMPROVEMENT IN TECHNOLOGY




- technology improves from $A^l$ to $A^h > A^l$
- population is at steady state $L^*$, so output per worker increases from $y^* = [A^l X/L^*]^\alpha$ to $y^h = [A^h X/L^*]^\alpha$
- as output per worker is above $y^*$, there is more food per household than in steady state, so households have more children
- population is increasing over time, so output per worker is falling over time

# THE MALTHUSIAN TRAP

- in the short run, an increase in technology raises output per worker
  - because land and working population are determined at the time of the shock
- but in the long run, technology has no effect on output per worker and thus consumption per capita
  - higher technology implies higher population, which absorbs the higher output produced with the better technology
- the short-run increase in output per worker temporarily leads to more children, which raises population and eventually reduces output per worker

# THE MALTHUSIAN TRAP

- what happens for technological improvements also occurs for the discovery of new arable land
    - a land expansion raises output per worker in the short run
- but in the long run, new land has no effect on output per worker and thus consumption per capita
    - more land implies higher population, which absorbs the higher output produced with larger amount of land
- the model generates a Malthusian trap: standards of living do not improve with better technology or more land
    - the only change is that population increases

# SUMMARY: THE MALTHUSIAN ERA

- the Malthusian model describes well the Malthusian era that prevailed before the Industrial Revolution (in 19th century)
- output per capita fluctuates around a subsistence level
- technological progress and land expansion lead to
  - temporary increase in output per capita
  - no effect on output per capita in the long run
  - increase in population in the long run
- the Malthusian model is a model of population growth, not a model of growth in standards of living

# EMPIRICAL IMPLICATIONS OF THE MALTHUSIAN MODEL

- regions with high technology or a lot of arable land have
  - high population density
  - but same output per capita
- variations in technological advancement across countries will be reflected in variations in population density:
  - in line with effect of land quality —> higher population density, but no effect on income per capita