

LECTURE 25

SOLOW | PRODUCTION & SAVING

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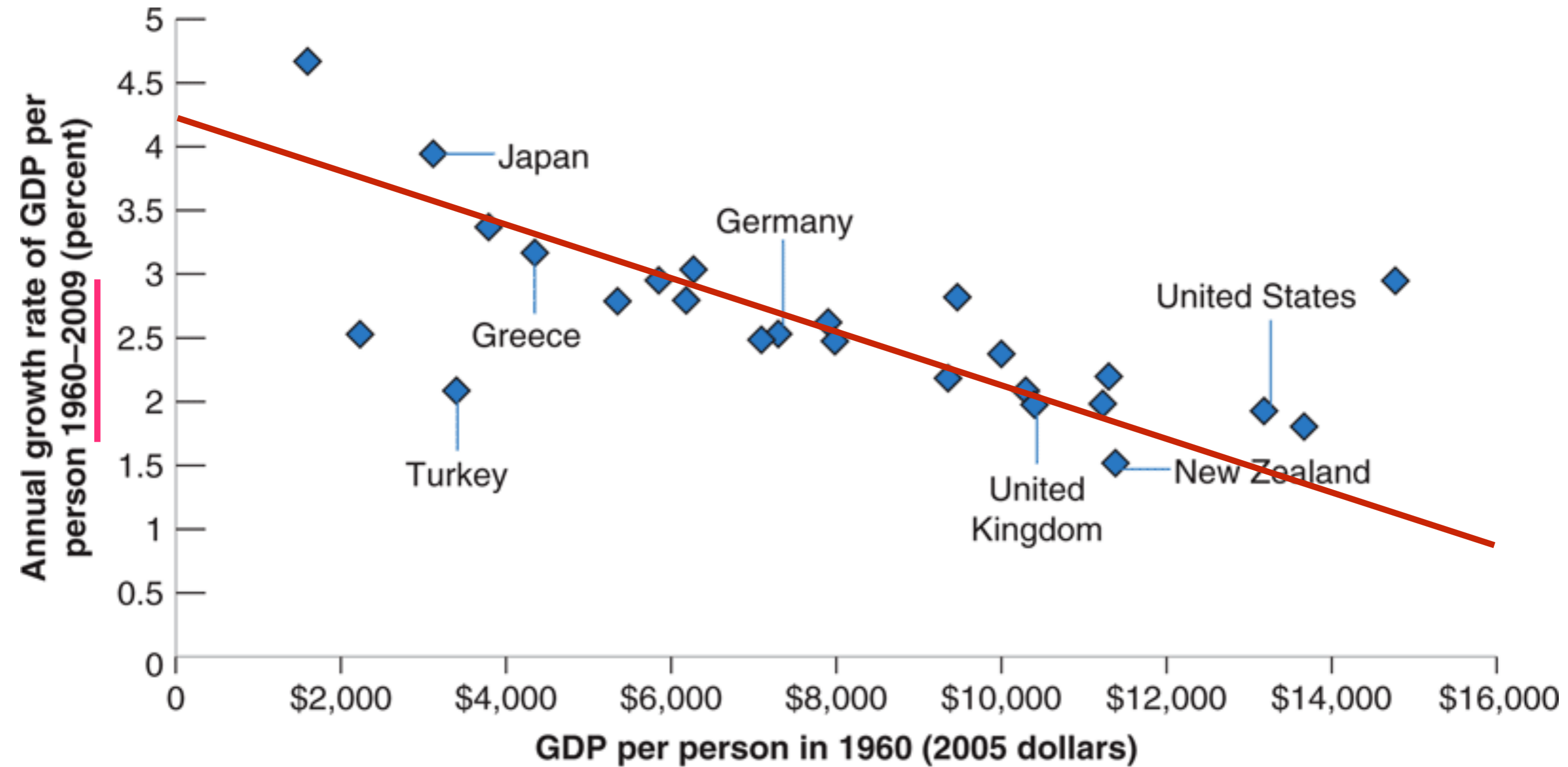
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US GROWTH AFTER INDUSTRIAL REVOLUTION

- US real GDP / person: $GDP(2014) = 9 \times GDP(1890)$
 - due to better technology: inventions, production and management techniques, infrastructure, legal and political institutions
- the Malthusian model predicts that technology advancement leads to higher population but same standards of living (Malthusian trap)
 - root cause: the amount of land is fixed
- to explain how technological progress leads to growth in output per worker, we introduce the Solow model of growth
 - production relies on the capital stock, which grows with technology

CONVERGENCE FOR OECD COUNTRIES



Source: See Table 10-1.

PRODUCTION FUNCTION

- aggregate production function: $Y = F(K,N)$
 - Y: output
 - K: capital
 - N: labor
- the technology level in the economy determines the level of $F(K,N)$
 - in more technologically advanced economies, for a given K and N, $F(K,N)$ is higher

PROPERTIES OF PRODUCTION FUNCTION

- $F(K,N)$ is increasing in K and N
- $F(K,N)$ has constant returns to scale
 - $F(b \times K, b \times N) = b \times F(K,N)$ for any scalar b
- $F(K,N)$ has decreasing returns to capital
 - $F(K,N)$ is concave in K : for a given N , increases in K lead to smaller and smaller increases in $F(K,N)$
- $F(K,N)$ has decreasing returns to labor
 - $F(K,N)$ is concave in N : for a given K , increases in N lead to smaller and smaller increases in $F(K,N)$

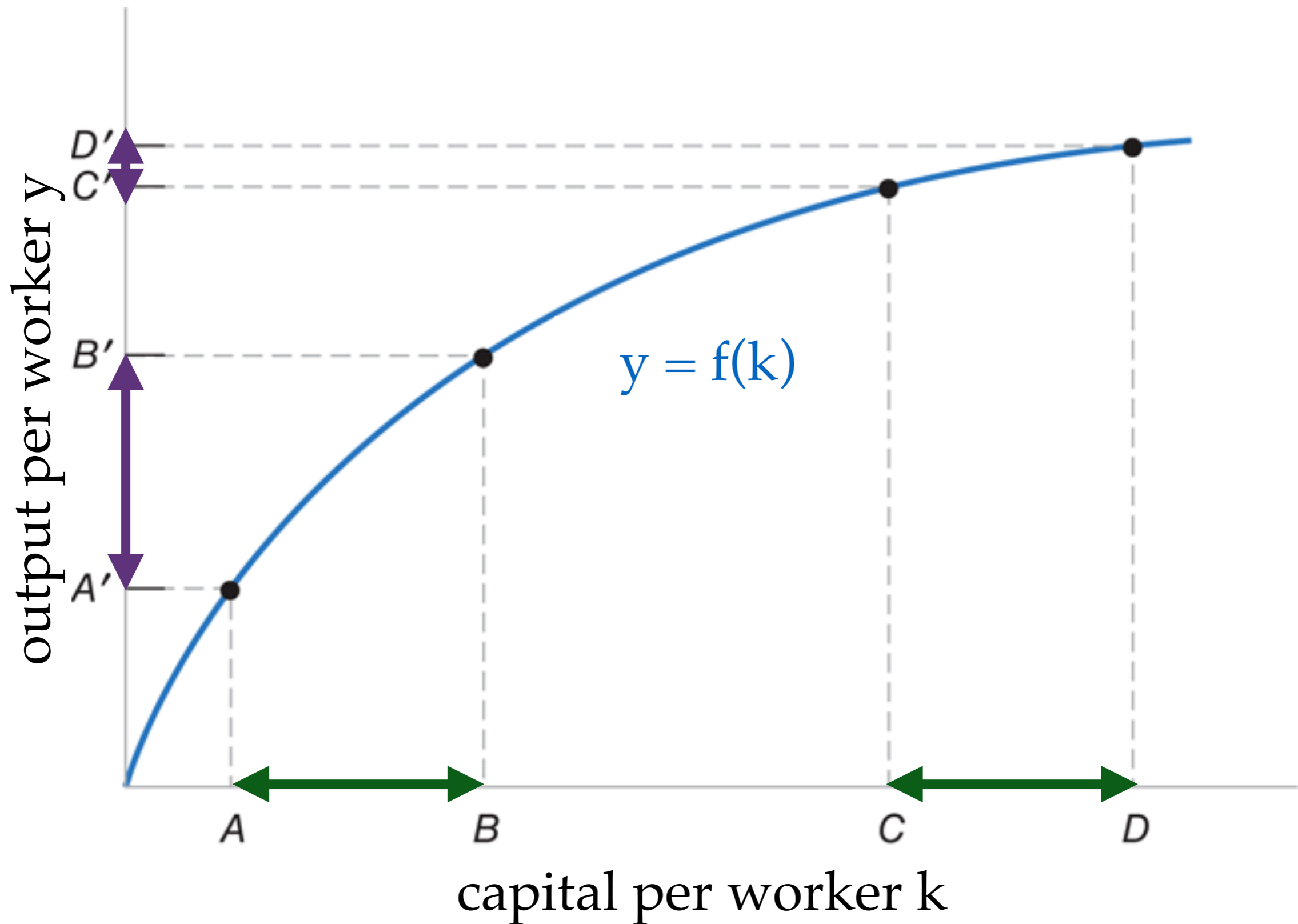
OUTPUT AND CAPITAL PER WORKER

- by constant returns to scale, there is a simple relation between
 - output per worker $y = Y/N$
 - and capital per worker $k = K/N$
- $y = Y/N = F(K,N)/N = F(K/N, N/N) = F(k,1) = f(k)$
 - the function f is defined by $f(x) = F(x,1)$
- $f(k)$ is increasing in k because $F(K,N)$ is increasing in K
- $f(k)$ is concave in k because $F(K,N)$ has decreasing returns to capital

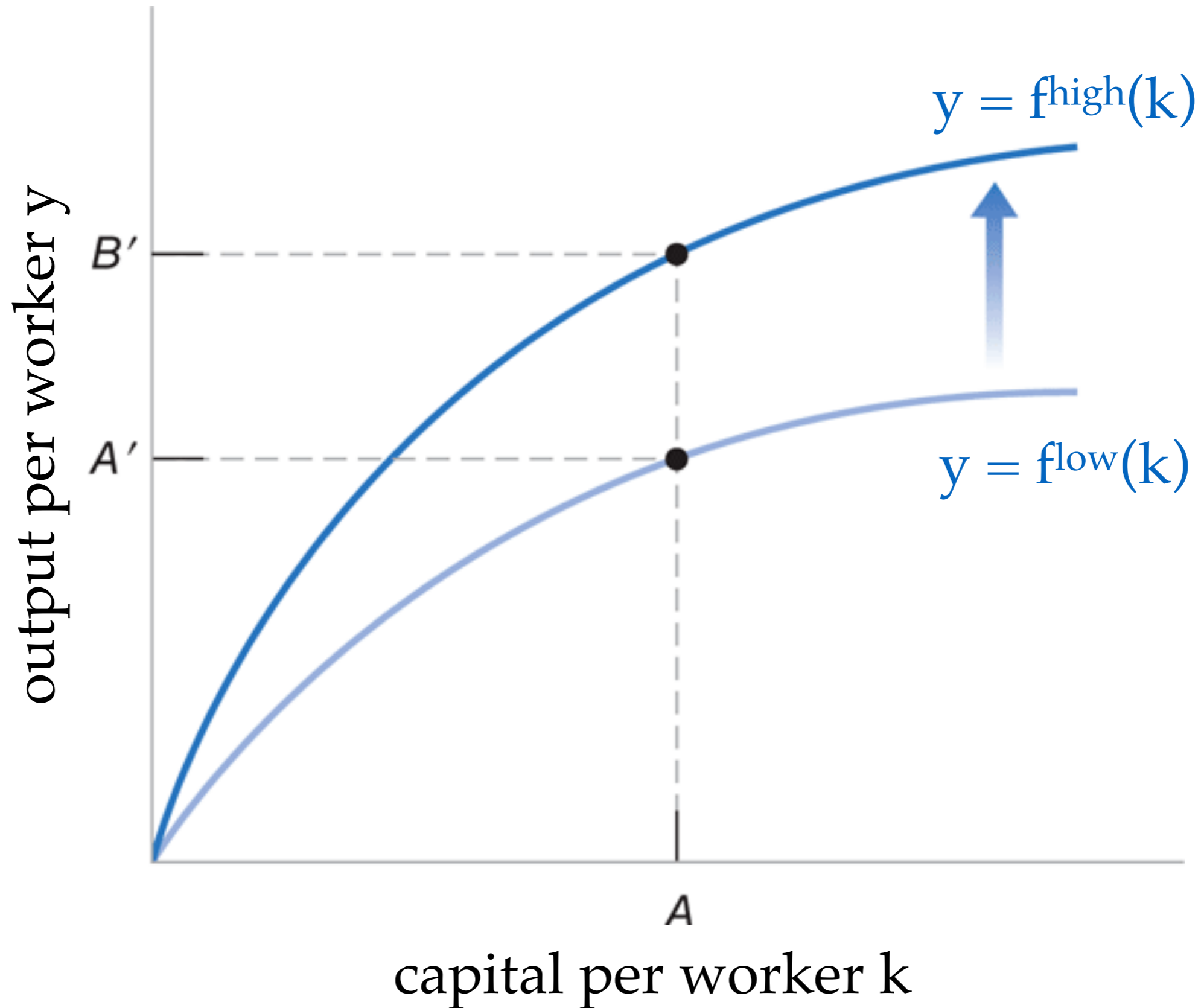
TWO SOURCES OF GROWTH

- output per worker is $y = f(k)$
 - output per worker measures standards of living
- growth of output per worker comes from
 - capital accumulation: a higher k
 - movement along the production function
 - technological progress: a higher $f(\cdot)$
 - shift up of the production function

CAPITAL ACCUMULATION AND DECREASING RETURNS TO CAPITAL



TECHNOLOGY PROGRESS



SAVING BEHAVIOR

- assumption 1: closed economy (no exports or imports)
 - investment = private saving + public saving
- assumption 2: no public saving (government spending = tax revenues)
 - investment = private saving
- assumption 3: private saving S is proportional to income Y
 - $S(t) = s \times Y(t)$
 - $s > 0$ is a parameter measuring the saving rate
- conclusion: output Y and investment I are related by $I(t) = s \times Y(t)$

LAW OF MOTION OF CAPITAL STOCK

- evolution of the capital stock is driven by:
 - investment I : increase in capital stock
 - depreciation $\delta \times K$: decrease in capital stock , because part of the capital stock becomes obsolete
 - $\delta > 0$ is the depreciation rate
- hence: $K(t+1) = K(t) + I(t) - \delta \times K(t)$
- investment comes from saving behavior:
 - $K(t+1) = (1 - \delta) \times K(t) + s \times Y(t)$

SAVING VERSUS DEPRECIATION

- simplifying assumption: employment N is constant
- we divide law of motion of K by N :
 - $K(t+1)/N = (1 - \delta) \times K(t)/N + s \times Y(t)/N$
- hence: $k(t+1) - k(t) = s \times y(t) - \delta \times k(t)$
 - the change in capital per worker = saving per worker minus depreciation per worker
 - if saving per worker $>$ depreciation per worker, capital per worker increases over time

LAW OF MOTION AND STEADY STATE OF CAPITAL PER WORKER

- we use production function to obtain law of motion:
 - $k(t+1) - k(t) = s \times f(k(t)) - \delta \times k(t)$
- law of motion: capital per worker today ($k(t)$) determines capital per worker tomorrow ($k(t+1)$)
- in steady state, capital per worker is constant:
 - investment per worker = depreciation per worker
 - steady-state capital k^* is such that $s \times f(k^*) = \delta \times k^*$